Faculty of Science, Technology, Engineering and Mathematics
MST224 Mathematical methods

MST224 diagnostic quiz

Am I ready to start on MST224?

The diagnostic quiz below is designed to help you to answer this question. This document also contains some advice on preparatory work that you may find useful before starting on MST224 (see below and pages 10–11).

The mathematical skills required for MST224 can be separated into two levels:

A those that are assumed but not discussed at all in the module;

B those that are reviewed in the *Unit 1* of the module.

The diagnostic quiz below is divided into corresponding sections A and B.

To be ready to start on MST224, you should be confident about level A topics. You should also have met level B topics before, and be able to handle them with the brief reminder provided in Unit 1 of the module.

Try the questions now, and then see the guidance on pages 10–11 of this booklet to see if you are ready for MST224.

Do contact your Student Support Team via StudentHome if you have any queries about your suitability for the module.

2.1

Diagnostic Quiz – Questions

LEVEL A

Question 1

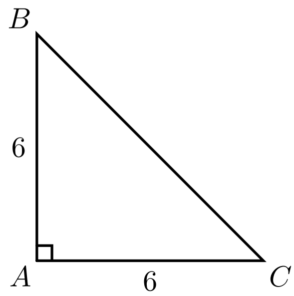
Using a calculator, give the values of each of the following to three decimal places:

(a) $\tan(1.2)$ (where 1.2 is in radians);

(b) $e^{-2.731}$;

(c) $\ln(4/27)$.

Question 2



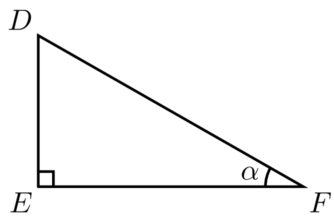
In the triangle BAC, the angle BAC is a right angle, and the sides AB and AC are each of length 6.

(a) Give each of the angles in triangle ABC in degrees and in radians.

(b) What is the length BC?

(c) What is the area of triangle ABC?

Question 3



(a) In the triangle DEF, the angle DEF is a right angle, and angle EFD is α. Write down each of $\cos\alpha$, $\sin\alpha$ and $\tan\alpha$ as ratios of sides in the triangle DEF.

(b) Give the values of $\cos(180°)$ and $\sin(270°)$.

Question 4

Solve for x each of the following equations.

(a) $3x + 4 = 10$

(b) $3(x + 3) - 7(x - 1) = 0$

(c) $\dfrac{2}{1+x} = \dfrac{3}{2-x}$

(d) $\sqrt{x^2 + 7} = 4$

Question 5

(a) Make t the subject of the equation

$$x = x_0 - \tfrac{1}{2}gt^2.$$

(b) Make x the subject of the equation

$$\sqrt{\frac{x-2}{x+3}} = t.$$

Question 6

Give the equation of the straight line passing through the points $y = 2$ when $x = 0$ and $y = 8$ when $x = 2$. What is the gradient of this line?

Question 7

If $y(x) = 3 + 2x - \sin(2x)$, what is $y\left(\frac{\pi}{2}\right)$?

Question 8

(a) Solve for y the equation

$$2y^2 - 4y + 1 = 0.$$

(b) Solve for λ the equation

$$\lambda^2 + 4\lambda + 4 = 0.$$

Question 9

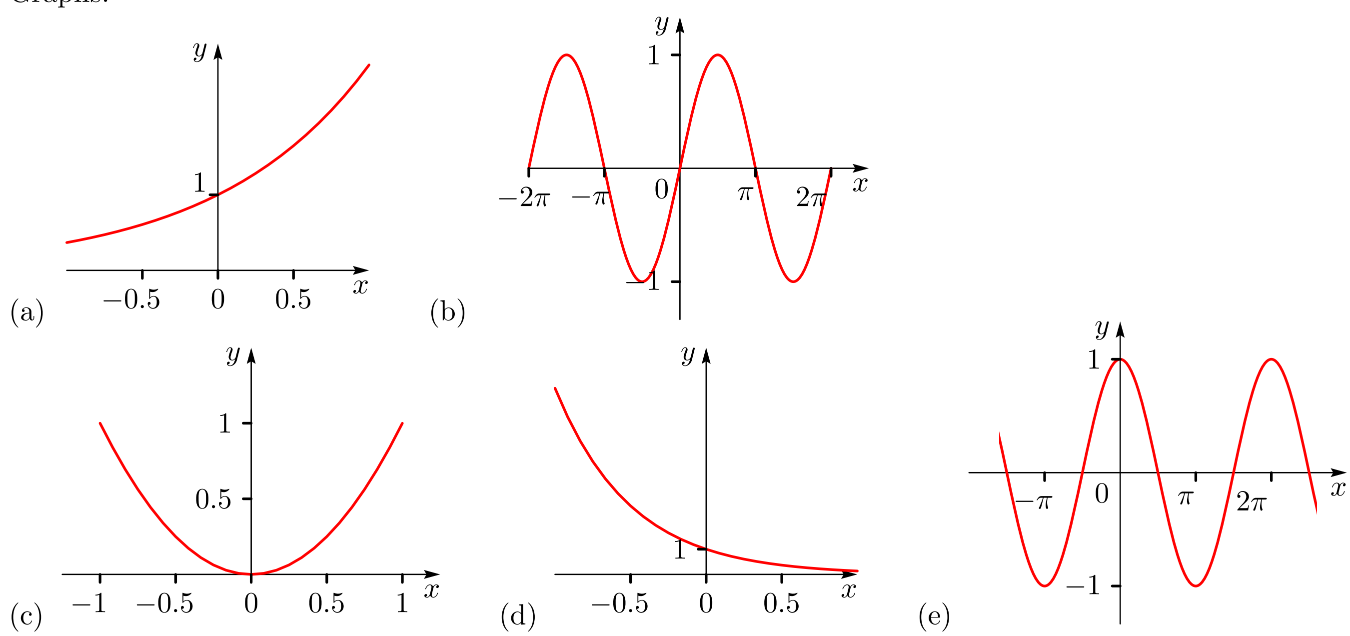
Solve the following simultaneous equations for x and y:

$$2x - y = 3,$$
$$3x + y = 2.$$

Question 10

Five graphs are given in parts (a)–(e) of the figure below. Each graph is that of one of the functions (i)–(v). Match each graph with the appropriate function.

Graphs:



Functions:

(i) $y(x) = e^{-x}$ (ii) $y(x) = e^{2x}$ (iii) $y(x) = \sin x$ (iv) $y(x) = \cos x$ (v) $y(x) = x^2$.

Question 11

Simplify each of the following.

(a) x^2x^5 (b) x^3/x^4 (c) $(x^2)^3$ (d) $9^{1/2}$

Question 12

If $|x-2| < 10^{-2}$, what range of values can x take?

Question 13

Solve for ν the equation below (where $m \neq 0$):

$$-\frac{m}{gr} = -\frac{\mu m}{\nu^2}.$$

LEVEL B

Question 14

Express $(e^{-2x} \times e^{3x})^2$ in the form e^y.

Question 15

Express $\frac{1}{2}\ln(25) + 3\ln(\frac{1}{2})$ in the form $\ln(y)$.

Question 16

Show that $y = \ln(2e^{-x/2})$ is the equation of a straight line. What is the gradient of this line?

Question 17

Solve for y the equation

$$\ln(y) = 2\ln(x) - 1.$$

Question 18

(a) What solutions for x has the equation $\sin x = 1$?

(b) What value does your calculator give for $\arcsin(1)$?

Question 19

What is $\cos^2\alpha + \sin^2\alpha$ (where α may be any real number)?

Question 20

Use the trigonometric identity $\cos(a+b) = \cos a \cos b - \sin a \sin b$, and particular values of a and b, to simplify $\cos(\pi + x)$.

Question 21

Find the local maxima and minima of the function

$$y(x) = 2x^3 - 3x^2 - 12x + 6.$$

Question 22

(a) Find $\dfrac{ds}{dt}$ where $s = 5e^{3t}$.

(b) Find $y'(t)$ where $y(t) = 3t^5 - 10\sqrt{t}$.

(c) Find $\dfrac{dz}{dx}$ where $z = 14\sin(x/8)$.

Question 23

Evaluate each of the following integrals.

(a) $\displaystyle\int (1 + 6x^3)\,dx$ (b) $\displaystyle\int_0^{\pi} \sin(3t)\,dt$

Question 24

Suppose that

$$-\frac{m}{gr} \geq -\frac{\mu m}{\nu^2},$$

where m, r, g and μ are positive.

(a) Rearrange this inequality by multiplying each side first by $-\nu^2$, then by gr/m.

(b) In terms of the other parameters, what is the largest value that ν can take?

Question 25

(a) (i) Find $y'(t)$ where $y(t) = t\sin(3t)$.

(ii) Find $\dfrac{dx}{dt}$ where $x = \ln(t^3 + 1)$.

(b) Find the velocity at time $t = 3$ of an object whose position at time t is given by $x(t) = e^{-2t}\cos(\frac{\pi}{3}t)$.

Question 26

(a) Use integration by substitution to find $\displaystyle\int x^2 \exp(2 + 3x^3)\,dx$.

(b) Use integration by parts to find $\displaystyle\int x \ln x\,dx$.

Question 27

Express the complex number $(1 + i)(3 + 2i)$ in the form $a + bi$.

Question 28

Find the imaginary part of the complex number $(e^{2+i\pi})^3$.

Solutions to questions

LEVEL A

Solution to Question 1

(a) 2.572

(b) 0.065

(c) −1.910

[See Revise and Refresh for MST224, Essential basics]

Solution to Question 2

(a) $\angle ABC = \angle ACB = 45^\circ = \frac{\pi}{4}$ radians.

$\angle BAC = 90^\circ = \frac{\pi}{2}$ radians.

(b) By Pythagoras's Theorem,

$$BC^2 = AB^2 + AC^2 = 6^2 + 6^2 = 72,$$

so

$$BC = \sqrt{72} = 6\sqrt{2}.$$

(c) The area of a triangle equals half its base times its height, so the area of triangle ABC is

$$\tfrac{1}{2} \times 6 \times 6 = 18 \text{ square units.}$$

[See Revise and Refresh for MST224, Trigonometry and Complex Numbers]

Solution to Question 3

(a) $\cos\alpha = \dfrac{EF}{DF}$, $\sin\alpha = \dfrac{DE}{DF}$, $\tan\alpha = \dfrac{DE}{EF}$.

(b) $\cos(180^\circ) = -1$, $\sin(270^\circ) = -1$.

[See Revise and Refresh for MST224, Trigonometry and Complex Numbers]

Solution to Question 4

(a) $3x + 4 = 10$

$3x = 6$

$x = 2$

(b) $3(x+3) - 7(x-1) = 0$

$3x + 9 - 7x + 7 = 0$

$-4x + 16 = 0$

$x = 4$

(c) $\frac{2}{1+x} = \frac{3}{2-x}$

$2(2-x) = 3(1+x)$

$4 - 2x = 3 + 3x$

$5x = 1$

$x = \frac{1}{5}$

(d) $\sqrt{x^2+7} = 4$

$x^2 + 7 = 4^2 = 16$

$x^2 = 9$

$x = \pm\sqrt{9}$

So $x = 3$ or $x = -3$.

[See Revise and Refresh for MST224, Essential basics]

Solution to Question 5

(a) $x = x_0 - \frac{1}{2}gt^2$

$gt^2 = 2(x_0 - x)$

$t = \pm\sqrt{\frac{2}{g}(x_0 - x)}$

(b) $\sqrt{\frac{x-2}{x+3}} = t$

$\frac{x-2}{x+3} = t^2$

$x - 2 = t^2(x+3) = t^2x + 3t^2$

$x(1-t^2) = 2 + 3t^2$

$x = \frac{2+3t^2}{1-t^2}$

[See Revise and Refresh for MST224, Essential basics]

Solution to Question 6

The equation of a straight line has the form

$$y = mx + c.$$

To satisfy the given conditions, the constants m and c must satisfy the equations

$$\begin{aligned} 2 &= c && (\text{since } y = 2 \text{ when } x = 0),\\ 8 &= 2m + c && (\text{since } y = 8 \text{ when } x = 2). \end{aligned}$$

Thus $c = 2$ and $m = 3$, so the required equation is

$$y = 3x + 2.$$

The gradient of this line is given by m, and so is 3.

[See Revise and Refresh for MST224, Functions and Graphs]

Solution to Question 7

$$\begin{aligned} y\left(\tfrac{\pi}{2}\right) &= 3 + 2 \times \tfrac{\pi}{2} - \sin\left(2 \times \tfrac{\pi}{2}\right)\\ &= 3 + \pi - \sin\pi\\ &= 3 + \pi - 0\\ &= 3 + \pi \end{aligned}$$

[See Revise and Refresh for MST224, Functions and Graphs]

Solution to Question 8

Use the formula for solving a quadratic equation.

(a)

$$y = \frac{4 \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)} = \frac{4 \pm \sqrt{8}}{4} = 1 \pm \tfrac{1}{2}\sqrt{2}$$

(b)

$$\lambda = \frac{-4 \pm \sqrt{16 - 16}}{2} = -2$$

(The expression $\lambda^2 + 4\lambda + 4$ is a perfect square, $(\lambda + 2)^2$.)

[See Revise and Refresh for MST224, Essential basics]

Solution to Question 9

Adding the equations gives $5x = 5$, so $x = 1$. Then the first equation gives $2 - y = 3$, so $y = -1$.

The solution is $x = 1$, $y = -1$.

[See Revise and Refresh for MST224, Functions and Graphs]

Solution to Question 10

The matching is as follows.

(a)(ii) (b)(iii) (c)(v) (d)(i) (e)(iv)

[See Revise and Refresh for MST224, Functions and Graphs]

Solution to Question 11

We use the rules for manipulating indices.

(a) $x^2 x^5 = x^{2+5} = x^7$

(b) $x^3/x^4 = x^{3-4} = x^{-1} (= 1/x)$

(c) $(x^2)^3 = x^{2\times 3} = x^6$

(d) $9^{1/2} = \sqrt{9} = 3$

[See Revise and Refresh for MST224, Essential basics]

Solution to Question 12

Recall that $|y|$ means y if $y \geq 0$, and $-y$ if $y < 0$. If $|x - 2| < 10^{-2}$, then $-10^{-2} < x - 2 < 10^{-2}$, so $2 - 10^{-2} < x < 2 + 10^{-2}$, i.e. $1.99 < x < 2.01$.

[See Revise and Refresh for MST224, Functions and Graphs]

Solution to Question 13

If $-\dfrac{m}{gr} = -\dfrac{\mu m}{\nu^2}$, then multiplying each side by $-\dfrac{\nu^2}{m}$ gives

$$\left(-\frac{\nu^2}{m}\right)\left(-\frac{m}{gr}\right) = \left(-\frac{\nu^2}{m}\right)\left(-\frac{\mu m}{\nu^2}\right),$$

i.e. $\dfrac{\nu^2}{gr} = \mu$. Hence $\nu = \pm\sqrt{\mu g r}$.

[See Revise and Refresh for MST224, Essential basics]

LEVEL B

Solution to Question 14

$(e^{-2x} \times e^{3x})^2 = (e^{3x-2x})^2 = (e^x)^2 = e^{2x}$

[See MST224 Unit 1, Subsection 1.4]

Solution to Question 15

$$\begin{aligned} \tfrac{1}{2}\ln(25) + 3\ln(\tfrac{1}{2}) &= \ln(\sqrt{25}\,) + \ln((\tfrac{1}{2})^3) \\ &= \ln(5) + \ln(\tfrac{1}{8}) \\ &= \ln(\tfrac{5}{8}) \end{aligned}$$

[See MST224 Unit 1, Subsection 2.2]

Solution to Question 16

Using the properties of exponentials and logarithms,

$$\ln(2e^{-x/2}) = \ln(2) + \ln(e^{-x/2}) = \ln(2) - \tfrac{1}{2}x,$$

so

$$y = \ln(2) - \tfrac{1}{2}x.$$

This is the equation of a straight line.

The gradient of the straight line is $-\frac{1}{2}$ (the coefficient of x).

[See MST224 Unit 1, Subsections 1.1 and 2.2]

Solution to Question 17

If $\ln(y) = 2\ln(x) - 1$, then taking exponentials of each side gives

$$\begin{aligned} \exp(\ln(y)) &= \exp(2\ln(x) - 1) \\ y &= \exp(\ln(x^2) - 1) \\ &= \exp(\ln(x^2))/\exp(1) \\ &= x^2/e. \end{aligned}$$

[See MST224 Unit 1, Subsection 2.2]

Solution to Question 18

(a) $\sin x = 1$ when $x = \frac{\pi}{2}$, or when x differs from $\frac{\pi}{2}$ by a multiple of 2π.

(b) My calculator gives $\arcsin(1) = 90$, but that is because it is working in degrees. If your calculator is working in radians (as it will need to be for MST224), then it should give

$$\arcsin(1) = 1.570796327 \quad (\text{i.e. } \tfrac{\pi}{2}).$$

[See MST224 Unit 1, Subsections 3.1 and 3.2]

Solution to Question 19

$\cos^2 \alpha + \sin^2 \alpha = 1.$

[See MST224 Unit 1, Subsection 3.3]

Solution to Question 20

We have

$$\begin{aligned}\cos(\pi + x) &= \cos\pi \cos x - \sin\pi \sin x \\ &= (-1)\cos x - (0)\sin x \\ &= -\cos x.\end{aligned}$$

[See MST224 Unit 1, Subsection 3.3]

Solution to Question 21

To find local maxima and minima, first find the stationary points, where $\dfrac{dy}{dx} = 0$.

Differentiating $y = 2x^3 - 3x^2 - 12x + 6$ gives

$$\frac{dy}{dx} = 6x^2 - 6x - 12.$$

So to find the stationary points, solve

$$6x^2 - 6x - 12 = 0,$$

i.e.

$$x^2 - x - 2 = 0.$$

To solve this quadratic equation, you can either use the formula, or factorize to obtain

$$(x - 2)(x + 1) = 0.$$

Thus there are stationary points at $x = 2$ and $x = -1$.

Now $\dfrac{d^2y}{dx^2} = 12x - 6.$

At $x = -1$, this is negative, so there is a local maximum at $x = -1$, of value $y = 13$.

At $x = 2$, this second derivative is positive, so there is a local minimum at $x = 2$, of value $y = -14$.

[See MST224 Unit 1, Subsection 5.3]

Solution to Question 22

(a) If $s = 5e^{3t}$, then

$$\frac{ds}{dt} = 5(3e^{3t}) = 15e^{3t}.$$

(b) If $y(t) = 3t^5 - 10\sqrt{t}$, then

$$y'(t) = 15t^4 - 5t^{-1/2}.$$

(c) If $z = 14\sin(x/8)$, then

$$\frac{dz}{dx} = \tfrac{14}{8}\cos(x/8) = \tfrac{7}{4}\cos(x/8).$$

[See MST224 Unit 1, Subsection 5.2]

Solution to Question 23

(a) This is an indefinite integral:

$$\begin{aligned}\int (1 + 6x^3)\, dx &= x + \tfrac{6}{4}x^4 + c \\ &= x + \tfrac{3}{2}x^4 + c,\end{aligned}$$

where c is an arbitrary constant.

(b) This is a definite integral:

$$\begin{aligned}\int_0^\pi \sin(3t)\, dt &= \left[-\tfrac{1}{3}\cos(3t)\right]_0^\pi \\ &= -\tfrac{1}{3}(\cos(3\pi) - \cos(0)) \\ &= -\tfrac{1}{3}(-1 - 1) = \tfrac{2}{3}.\end{aligned}$$

[See MST224 Unit 1, Subsection 6.2]

Solution to Question 24

We have

$$-\frac{m}{gr} \geq -\frac{\mu m}{\nu^2}.$$

(a) The quantity $-\nu^2$ is negative, so on multiplying both sides by $-\nu^2$, we must reverse the inequality:

$$\frac{m}{gr}\nu^2 \leq \mu m.$$

Then (since gr/m is positive)

$$\nu^2 \leq \mu g r.$$

(b) The largest value that ν can take is $\sqrt{\mu g r}$.

[See MST224 Unit 1, Subsection 5.2]

Solution to Question 25

(a) **(i)** To differentiate $y(t) = t\sin(3t)$, use the product rule. We obtain

$$y'(t) = \sin(3t) + 3t\cos(3t).$$

(ii) To differentiate $x = \ln(t^3 + 1)$, use the composite (or 'function of a function') rule. We obtain

$$\frac{dx}{dt} = 3t^2 \times \frac{1}{t^3 + 1} = \frac{3t^2}{t^3 + 1}.$$

(b) To find the velocity $v(t)$ of the object, we differentiate the expression for its position, i.e.

$$\begin{aligned} v(t) &= \frac{dx}{dt} \\ &= -2e^{-2t}\cos(\tfrac{\pi}{3}t) - \tfrac{\pi}{3}e^{-2t}\sin(\tfrac{\pi}{3}t) \\ &= -e^{-2t}(2\cos(\tfrac{\pi}{3}t) + \tfrac{\pi}{3}\sin(\tfrac{\pi}{3}t)). \end{aligned}$$

So at time $t = 3$, the object's velocity is

$$\begin{aligned} v(3) &= -e^{-6}(2\cos\pi - \tfrac{\pi}{3}\sin\pi) \\ &= -e^{-6}(2(-1) + \tfrac{\pi}{3}(0)) \\ &= 2e^{-6} \\ &\simeq 0.005. \end{aligned}$$

[See MST224 Unit 1, Subsection 5.2]

Solution to Question 26

(a) Integration by substitution uses the formula

$$\int f(u)\frac{du}{dx}\,dx = \int f(u)\,du.$$

With $u(x) = 2 + 3x^3$, we have $\dfrac{du}{dx} = 9x^2$, and then

$$\begin{aligned} \int x^2\exp(2+3x^3)\,dx &= \int \tfrac{1}{9}\frac{du}{dx}\exp u\,dx \\ &= \tfrac{1}{9}\int \exp u\,\frac{du}{dx}\,dx \\ &= \tfrac{1}{9}\int \exp u\,du \\ &= \tfrac{1}{9}\exp u + c \\ &= \tfrac{1}{9}\exp(2+3x^3) + c, \end{aligned}$$

where c is an arbitrary constant.

(b) Integration by parts uses the formula

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx.$$

With $f(x) = \ln x$ and $g(x) = \frac{1}{2}x^2$, we have $f'(x) = 1/x$ and $g'(x) = x$, and then

$$\begin{aligned} \int x\ln x\,dx &= \tfrac{1}{2}x^2\ln x - \int \tfrac{1}{2}x\,dx \\ &= \tfrac{1}{2}x^2\ln x - \tfrac{1}{4}x^2 + c, \end{aligned}$$

where c is an arbitrary constant.

[See MST224 Unit 1, Subsection 6.3]

Solution to Question 27

Since $i^2 = -1$, we have

$$\begin{aligned} (1+i)(3+2i) &= 3 + 2i + 3i + 2i^2 \\ &= 3 + 5i - 2 \\ &= 1 + 5i. \end{aligned}$$

[See MST224 Unit 1, Subsection 4.1]

Solution to Question 28

We have

$$\begin{aligned} (e^{2+i\pi})^3 &= e^{(2+i\pi)\times 3} \\ &= e^{6+3i\pi} \\ &= e^6(\cos(3\pi) + i\sin(3\pi)). \end{aligned}$$

The imaginary part of this is

$$e^6\sin(3\pi) = 0.$$

[See MST224 Unit 1, Subsection 4.2]

Resources to prepare for MST224

You can revise and refresh the topics needed for MST224, using the resources available, to all students registered with the university, on the Mathematics and Statistics study site (learn2.open.ac.uk/site/S-MATHS) under DISCOVER ... *Discover your module*.

The topics covered are:

- essential basics
- functions and Graphs
- trigonometry and Complex numbers
- vectors and Matrices
- differentiation
- integration

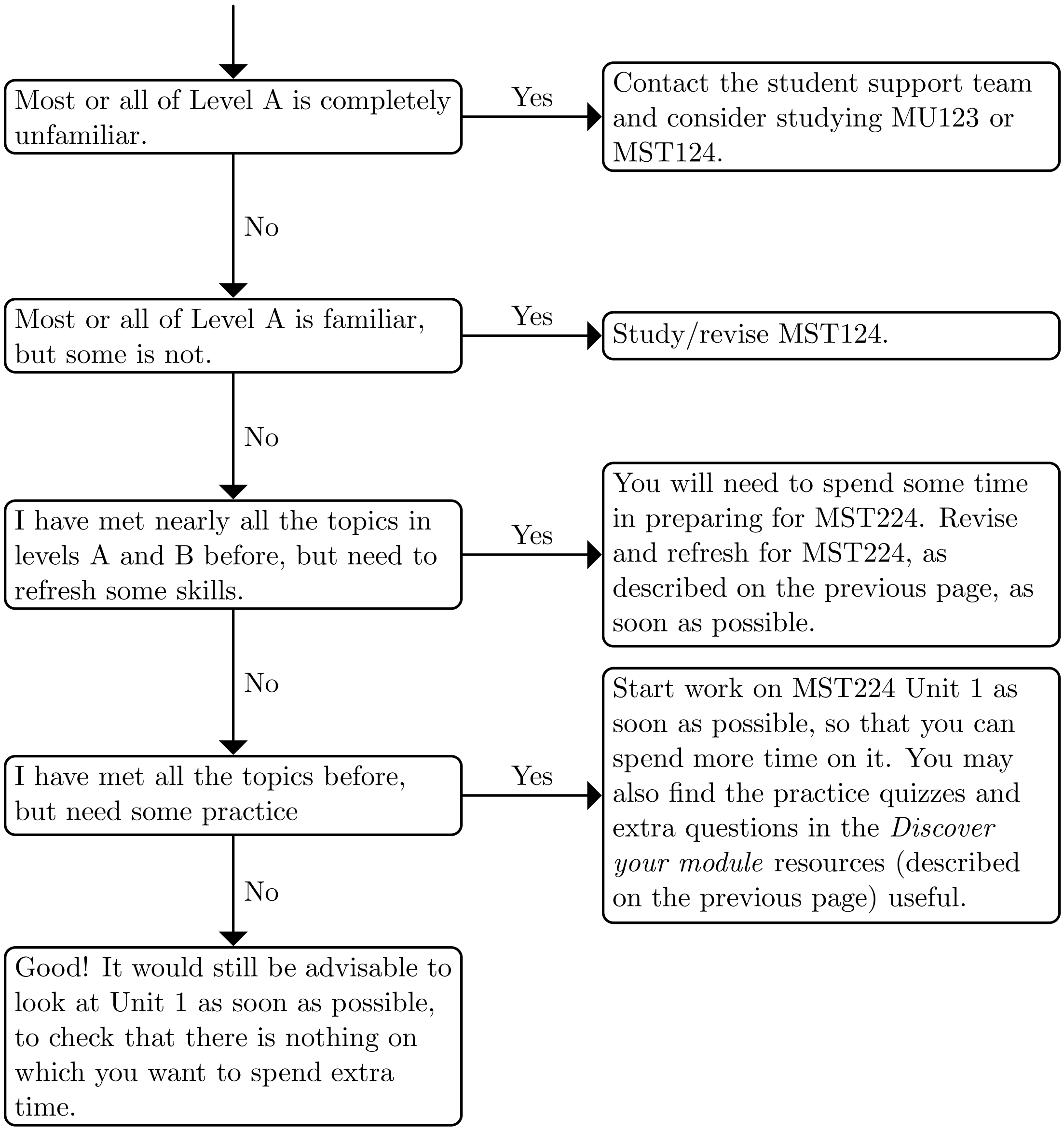
Unit 1 of MST224, also available at the above site, reviews a number of topics used in MST224, but does not introduce these topics from scratch; rather, it provides a reminder of them, and an opportunity to refresh your memory and practise techniques. Topics covered include:

- standard functions, such as linear, quadratic, exponential and logarithm functions, and algebraic manipulations involving these
- trigonometric functions and identities involving these
- complex numbers
- differentiation
- integration

Later units of MST224 expect you to be proficient in the ideas and methods covered in Unit 1, particularly the use of the various standard functions, including the trigonometric functions, and manipulation of expressions involving them, differentiation, and integration using the table of standard integrals given in the module Handbook.

What should I do to prepare for MST224?

Use of the following flowchart in connection with the diagnostic quiz should help you to decide whether MST224 is an appropriate course for you, and what you should do by way of preparation before the course starts.



Do contact your Student Support Team via StudentHome if you have any queries about your suitability for the module.