Open University Mathematics MSc Entry Test

This test is designed to help you check if you have sufficient mathematical background to enrol on the Mathematics MSc programme.

An online version of this test is available from
`https://learn1.open.ac.uk/course/view.php?id=100193&cmid=22153`

Please take the online version if you are able to do so. If you are unable to take the online version, please contact Maths-MSc@open.ac.uk.

You may take the test as many times as you wish before the end-date you have been given. The highest score you obtain in your attempts will be used to assess your preparedness for the Mathematics MSc programme. When you have completed your attempts, please email Maths-MSc@open.ac.uk by the end-date giving your highest score, which can then be confirmed by the admissions team.

We do not supply solutions for this test. There is an "Are you ready for the MSc in Mathematics" quiz available at
`https://learn1.open.ac.uk/course/view.php?id=100193&cmid=22037`
which has both solutions and advice on which modules will help you fill in any gaps.

Although the test is designed in a multiple-choice format you should be aware that for many of the questions you will need to do a pencil and paper calculation. **This version of the test is principally to help you work through the problems offline.**

If, after working through the test, you would like to discuss your study plan then you should contact Maths-MSc@open.ac.uk.

The test begins on the following page. There are 25 questions, each of which is worth 1 mark out of 25.

Answer each of the following. Select one of (a), (b), (c) and (d).

1. Suppose that the function $f : \mathbb{N} \to \mathbb{N}$ is defined recursively by:

$$f(n) = 2f(n-1) + 1; f(1) = 3.$$

Which of the following functions is equal to f?

(a) $f(n) = 2^{n+2} - 5$
(b) $f(n) = 2^{n-1} + 3$
(c) $f(n) = 2^n + 2n - 1$
(d) $f(n) = 2^{n+1} - 1$

2. Suppose that $a, b > 0$ and that $a + b = 2|a - b|$. Which of the following statements is true? Note: the arithmetic mean of a, b is $\dfrac{a+b}{2}$, the geometric mean of a, b is $\sqrt{ab}$.

(a) The arithmetic mean of a and b is $2b$.
(b) The arithmetic mean of a and b is $2a$.
(c) If $a > b$ then the geometric mean of a and b is $\sqrt{3}b$.
(d) The geometric mean of a and b is irrational.

3. Let $f(x) = \begin{cases} x \sin\left(\dfrac{1}{x}\right), & x > 0, \\ 0, & x = 0, \\ -\dfrac{x}{1+x^2}, & x < 0. \end{cases}$

Which of the following statements is true?

(a) The function f is differentiable at 0.
(b) The function f is continuous at 0.
(c) The function f is not differentiable at π.
(d) The function f is unbounded.

4. Let $F(x) = \int_0^x \sec t \, dt$. Which of the following statements is true?

(a) $F'(x) = \sec x \tan x$
(b) $F'(x) = \cos x$
(c) $F'(x) = \sec x$
(d) $F'(x) = \ln |\cos x|$

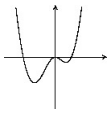
5. Given $x = r\cos\theta$ and $y = r\sin\theta$, which of the following expressions gives $\partial\theta/\partial x$.

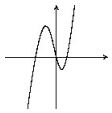
   (a) $-y$

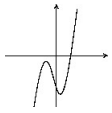
   (b) $\dfrac{y}{r^2}$

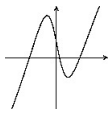
   (c) $-\dfrac{y}{r^2}$

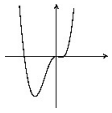
   (d) $\dfrac{-1}{y}$

6. Suppose that  shows the graph of $f(x)$. Which of the following could be the graph of $f'(x)$?

   (a) 

   (b) 

   (c) 

   (d) 

7. Which of the following is the coefficient of $(y-1)^2$ in the Taylor series about $x = 2,\ y = 1$ for the function

$$f(x, y) = x^3 + y^2 + \exp(xy)?$$

   (a) 0

   (b) $2 + 2\exp(2)$

   (c) $1 + 2\exp(2)$

   (d) $3\exp(2)$

8. Which of the following is given by $\int \tan^3 x \, dx$?

   (a) $\dfrac{\sin^2 x \tan^2 x}{4} + c$

   (b) $\ln|\cos x| - \tan^2 x + c$

   (c) $\dfrac{1}{2}\tan^2 x + \ln|\cos x| + c$

   (d) $\dfrac{\sin^4 x}{4} + c$

9. Which of the following is the solution, for a, of $\displaystyle\int_1^\infty \frac{a}{x(2x+a)}\,dx = 1$ where $a > -2$?

   (a) $2(e-1)$

   (b) 0

   (c) $2(e+1)$

   (d) $\dfrac{1}{2e} - 2$

10. Which of the following is the general solution of $\dfrac{dy}{dx} = \dfrac{y}{x} - \tan\left(\dfrac{y}{x}\right)$? Hint: use the substitution $y = xu(x)$.

    (a) $y = x \arcsin(c/x)$

    (b) $y = \arcsin(cx)$

    (c) $y = x \arcsin(c + x)$

    (d) $x = \dfrac{1}{2}\left(\dfrac{y}{x}\right)^2 - \ln\left|\cos\dfrac{y}{x}\right| + c$

11. Which of the following is the value of the integral $\displaystyle\int_C \frac{2}{(2z-1)(z-2)}dz$, where C is the circle $|z| = 1$?

    (a) $\dfrac{2\pi i}{3}$

    (b) -3

    (c) $\dfrac{3\pi i}{2}$

    (d) $-\dfrac{4}{3}\pi i$

12. Which of the following shows the curve defined parametrically by $f(t) = (\cos t, -\sin t), \pi \leq t \leq \frac{3\pi}{2}$?

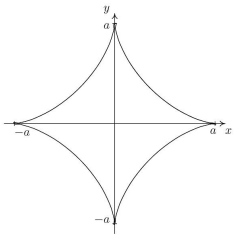
(a) 

(b) 

(c) 

(d) 

13. The length s of a curve defined parametrically by $(x(t), y(t))$ with end-points at $t = t_A$ and t_B is given by $s = \displaystyle\int_{t_A}^{t_B} \left\{ \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \right\}^{1/2} dt.$



Which of the following is the length of the circumference of the astroid defined for $a > 0$ by $x = a\cos^3 t,\ y = a\sin^3 t,\ 0 \leq t \leq 2\pi$?

(a) 0

(b) $2\sqrt{3a}$

(c) $3\sqrt{3a}$

(d) $6a$

14. Let

$$x(y) = c \int_{y/c}^{1} \frac{w^n}{\sqrt{1 - w^{2n}}}\, dw, \tag{1}$$

where n is a positive integer, $y(0) = c$ and $x(a) = A$ for constant c, a and A all greater than zero.

By setting $w^n = \sin z$, determine which of the following is equivalent to Equation (1) at the point (a, A)?

(a) $a = \dfrac{1}{n} c \displaystyle\int_{\frac{A}{c}}^{1} (\sin\phi)^{1/n} d\phi$

(b) $a = \dfrac{1}{n} c \displaystyle\int_{\frac{A}{c}}^{1} (1 - u^2)^{\frac{2n-1}{2n}} du$

(c) $\dfrac{a}{A} = \dfrac{1}{n} \left(\dfrac{1}{\sin z}\right)^{1/n} \displaystyle\int_{z}^{\pi/2} \tan\phi\, d\phi$ where $c^n = \dfrac{A^n}{\sin z}$

(d) $\dfrac{a}{A} = \dfrac{1}{n} \left(\dfrac{1}{\sin z}\right)^{1/n} \displaystyle\int_{z}^{\pi/2} (\sin\phi)^{1/n} d\phi$ where $c^n = \dfrac{A^n}{\sin z}$

15. Which of the following is the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4y = \sin(2x) \ ?$$

(a) $y = A\sin(2x) + B\cos(2x) - \dfrac{x}{4}\cos(2x)$

(b) $y = A\sin(2x) + B\cos(2x) - \dfrac{x}{4}\sin(2x)$

(c) $y = A\sin(2x) + B\cos(2x)$

(d) $y = (A + Bx)e^{-2x} - \dfrac{x}{4}\cos(2x)$

16. Which of the following is the particular solution to the equation in Question 15 when $y(0) = \ 1, y'(0) = -\dfrac{1}{4}$?

(a) $y = -\sin(2x) - \dfrac{x}{4}\cos(2x)$

(b) $y = -\dfrac{x}{4}\cos(2x)$

(c) $y = -\cos(2x)$

(d) $y = \cos(2x) - \dfrac{x}{4}\cos(2x)$

17. Make the change of variables $r = x + vt, s = x - vt$ in the wave equation

$$\frac{\partial^2 F}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 F}{\partial t^2} = 0.$$

Which of the following statements is correct?

(a) $\dfrac{\partial F}{\partial x} = \dfrac{\partial F}{\partial r} + \dfrac{\partial F}{\partial s}, \quad \dfrac{\partial F}{\partial t} = v\dfrac{\partial F}{\partial r} - v\dfrac{\partial F}{\partial s}$

(b) $\dfrac{\partial F}{\partial x} = \dfrac{\partial F}{\partial r} - \dfrac{\partial F}{\partial s}, \quad \dfrac{\partial F}{\partial t} = v\dfrac{\partial F}{\partial r} + v\dfrac{\partial F}{\partial s}$

(c) $\dfrac{\partial F}{\partial x} = \dfrac{\partial F}{\partial r}, \quad \dfrac{\partial F}{\partial t} = -v\dfrac{\partial F}{\partial s}$

(d) $\dfrac{\partial F}{\partial x} = v\dfrac{\partial F}{\partial r} + v\dfrac{\partial F}{\partial s}, \quad \dfrac{\partial F}{\partial t} = v\dfrac{\partial F}{\partial r} - v\dfrac{\partial F}{\partial s}$

18. Which of the following is the general solution of the wave equation in Question 17?

(a) $F = f(x - vt) + k$

(b) $F = k + g(x + vt)$

(c) $F = f(x - vt) + g(x + vt)$

(d) $F = x + vt + k$

19. Which of the following is the inverse of the matrix $\begin{pmatrix} a & -b \\ a & b \end{pmatrix}$?

(a) $\begin{pmatrix} b & b \\ -a & a \end{pmatrix}$

(b) $\dfrac{1}{ab}\begin{pmatrix} -b & -b \\ a & -a \end{pmatrix}$

(c) $\dfrac{1}{2ab}\begin{pmatrix} b & b \\ -a & a \end{pmatrix}$

(d) $\begin{pmatrix} a & -a \\ b & b \end{pmatrix}$

20. For what value (or values) of k is $(1, -1, k, 3)$ a linear combination of $(1, 0, 1, 0)$, $(0, 1, 0, 0)$ and $(0, 1, 0, 1)$?

(a) 0

(b) no values

(c) 1

(d) all values

21. Which of the following sets of vectors is an orthogonal basis for $\mathbb{R}^3$?

(a) $\{(1,0,0),(1,1,0),(0,0,1)\}$

(b) $\{(1,0,0),(1,-1,0),(0,0,1)\}$

(c) $\{(1,0,1),(1,0,-1),(0,1,0)\}$

(d) $\{(1,0,0),(1,1,0),(1,1,1)\}$

22. How many integers between 1 and 500 are divisible by 30 but not by 16?

(a) 15

(b) 16

(c) 14

(d) 13

23. The product of the eigenvalues of the matrix $\begin{pmatrix} i & 2 \\ -i & 2-i \end{pmatrix}$ is

(a) 2

(b) $4i+1$

(c) 0

(d) $2+2i$

24. Consider the following system of linear equations.

$$\begin{array}{rcrcrcr} x & + & 2y & - & z & = & 1 \\ -x & - & 2y & + & 3z & = & 0 \\ x & + & y & + & z & = & -2 \end{array}$$

Which of the following statements is true?

(a) The system of equations has a unique solution.

(b) The system of equations has no solution.

(c) The system of equations has an infinite number of solutions.

25. Which of the following is the coefficient of x^3 in the expansion of $(1-x^2)^2(1+x)^8$?

(a) 0

(b) -16

(c) 16

(d) 40

END OF TEST