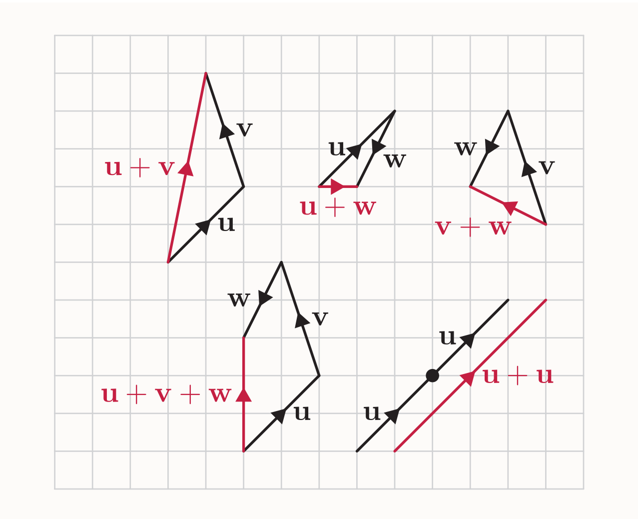
Solution to Activity 23

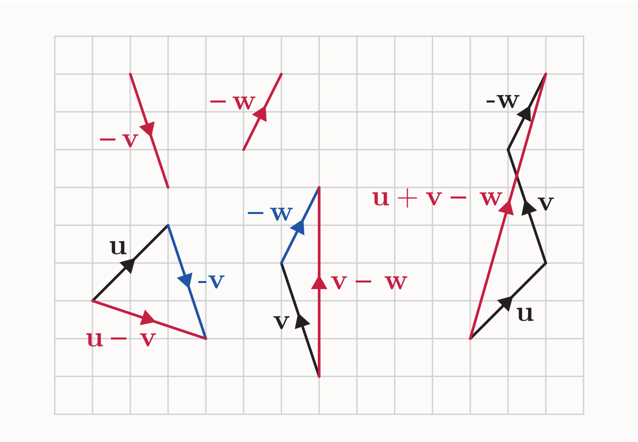
(a) The vector $\mathbf{f}$ is equal to the vector $\mathbf{a}$.

(b) The vector $\mathbf{d}$ is equal to the vector $\overrightarrow{PQ}$.

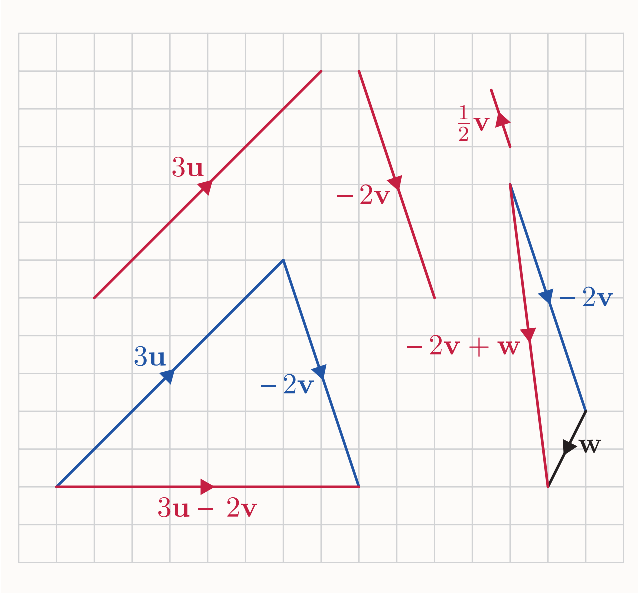
Solution to Activity 24



Solution to Activity 25



Solution to Activity 26



Solution to Activity 27

(a) The velocity of a wind of 70 knots blowing from the north-east is represented by $2\mathbf{v}$.

(b) The velocity of a wind of 35 knots blowing from the south-west is represented by $-\mathbf{v}$.

Solution to Activity 28

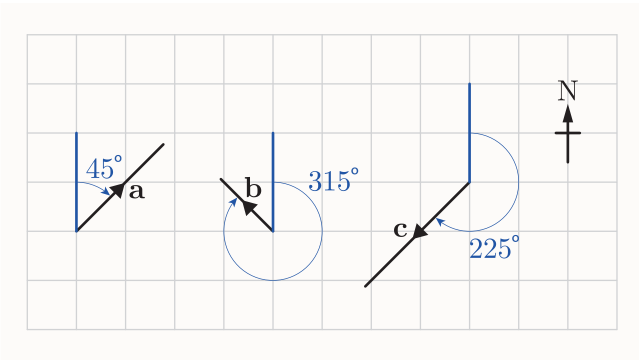
(a)
$$\begin{aligned} 4(\mathbf{a}-\mathbf{c}) + 3(\mathbf{c}-\mathbf{b}) + 2(2\mathbf{a}-\mathbf{b}-3\mathbf{c}) &= 4\mathbf{a}-4\mathbf{c}+3\mathbf{c}-3\mathbf{b}+4\mathbf{a}-2\mathbf{b}-6\mathbf{c} \\ &= 8\mathbf{a}-5\mathbf{b}-7\mathbf{c} \end{aligned}$$

(b) (i)
$$\begin{aligned} 4\mathbf{x} &= 7\mathbf{a}-2\mathbf{b} \\ \mathbf{x} &= \tfrac{7}{4}\mathbf{a}-\tfrac{1}{2}\mathbf{b} \end{aligned}$$

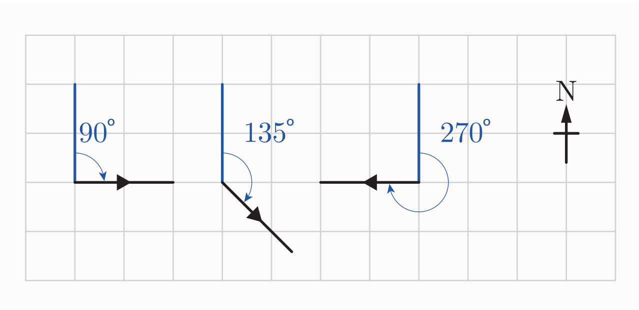
(ii)
$$\begin{aligned} 5\mathbf{x} &= 2(\mathbf{a}-\mathbf{b}) - 3(\mathbf{b}-\mathbf{a}) \\ &= 2(\mathbf{a}-\mathbf{b}) + 3(\mathbf{a}-\mathbf{b}) \\ &= 5(\mathbf{a}-\mathbf{b}) \\ \mathbf{x} &= \mathbf{a}-\mathbf{b} \end{aligned}$$

Solution to Activity 29

(a) The bearing of $\mathbf{a}$ is 45°, the bearing of $\mathbf{b}$ is 315°, and the bearing of $\mathbf{c}$ is 225°, as shown below.

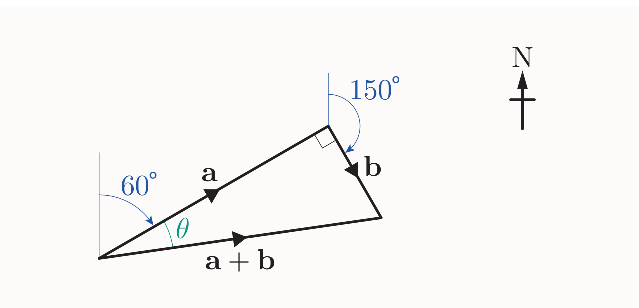


(b)



Solution to Activity 30

Represent the first part of the motion by the vector $\mathbf{a}$, and the second part by the vector $\mathbf{b}$. Then the resultant displacement is $\mathbf{a}+\mathbf{b}$.



Since $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, and $|\mathbf{a}| = 5.3\,\text{km}$ and $|\mathbf{b}| = 2.1\,\text{km}$,

$$\begin{aligned} |\mathbf{a}+\mathbf{b}| &= \sqrt{|\mathbf{a}|^2+|\mathbf{b}|^2} \\ &= \sqrt{5.3^2+2.1^2} = \sqrt{32.5} \\ &= 5.70\ldots\ \text{km}. \end{aligned}$$

The angle marked θ is given by

$$\tan\theta = \frac{|\mathbf{b}|}{|\mathbf{a}|} = \frac{2.1}{5.3},$$

so

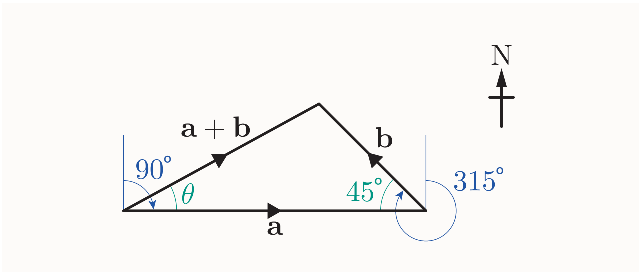
$$\theta = \tan^{-1}\left(\frac{2.1}{5.3}\right) = 22° \text{ (to the nearest degree).}$$

So the bearing of $\mathbf{a}+\mathbf{b}$ is $60° + 22° = 82°$ (to the nearest degree).

The resultant displacement of the yacht has magnitude 5.7 km (to 1 d.p.) and bearing 82° (to the nearest degree).

Solution to Activity 31

Represent the first part of the motion by the vector $\mathbf{a}$, and the second part by the vector $\mathbf{b}$. Then the resultant displacement is $\mathbf{a}+\mathbf{b}$.



The angle at the tip of $\mathbf{a}$ in the triangle above is $315° - 270° = 45°$, as shown.

We know that $|\mathbf{a}| = 40\,\text{cm}$ and $|\mathbf{b}| = 20\,\text{cm}$.

The magnitude of the resultant $\mathbf{a}+\mathbf{b}$ can be found by using the cosine rule:

$$|\mathbf{a}+\mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\times|\mathbf{a}|\times|\mathbf{b}|\times\cos 45°,$$

which gives

$$\begin{aligned} |\mathbf{a}+\mathbf{b}| &= \sqrt{|\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\times|\mathbf{a}|\times|\mathbf{b}|\times\cos 45°} \\ &= \sqrt{40^2+20^2-2\times 40\times 20\times\cos 45°} \\ &= 29.472\ldots\ \text{cm}. \end{aligned}$$

The angle marked θ can be found by using the sine rule:

$$\begin{aligned} \frac{|\mathbf{b}|}{\sin\theta} &= \frac{|\mathbf{a}+\mathbf{b}|}{\sin 45°} \\ \frac{20}{\sin\theta} &= \frac{29.472\ldots}{\sin 45°} \\ \sin\theta &= \frac{20\sin 45°}{29.472\ldots}. \end{aligned}$$

Now,

$$\sin^{-1}\left(\frac{20\sin 45°}{29.472\ldots}\right) = 28.67\ldots°,$$

so

$$\theta = 28.67\ldots°$$

or

$\theta = 180° - 28.67\ldots° = 151.32\ldots°.$

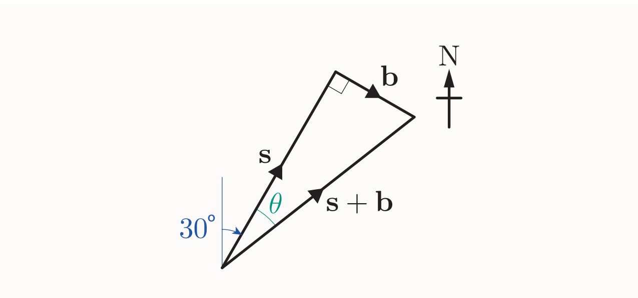
Since $|\mathbf{b}| < |\mathbf{a}+\mathbf{b}|$, we expect $\theta < 45°$, so the required solution is $\theta = 29°$ (to the nearest degree).

The bearing of $|\mathbf{a}+\mathbf{b}|$ is $90° - 29° = 61°$ (to the nearest degree).

So the resultant displacement of the grab has magnitude 29 cm (to the nearest cm) and bearing 61° (to the nearest degree).

Solution to Activity 32

Let $\mathbf{s}$ be the velocity of the ship, and let $\mathbf{b}$ be the velocity of the boy relative to the ship. Then the resultant velocity of the boy is $\mathbf{s}+\mathbf{b}$, as shown below.



We know that $|\mathbf{s}| = 10.0\,\mathrm{m\,s^{-1}}$ and $|\mathbf{b}| = 4.0\,\mathrm{m\,s^{-1}}$. Since the triangle is right-angled,

$$\begin{aligned}|\mathbf{s}+\mathbf{b}| &= \sqrt{|\mathbf{s}|^2+|\mathbf{b}|^2}\\ &= \sqrt{10^2+4^2}\\ &= \sqrt{116}\\ &= 10.77\ldots\ \mathrm{m\,s^{-1}}.\end{aligned}$$

The angle θ is given by

$$\tan\theta = \frac{|\mathbf{b}|}{|\mathbf{s}|} = \frac{4}{10} = \frac{2}{5}.$$

So

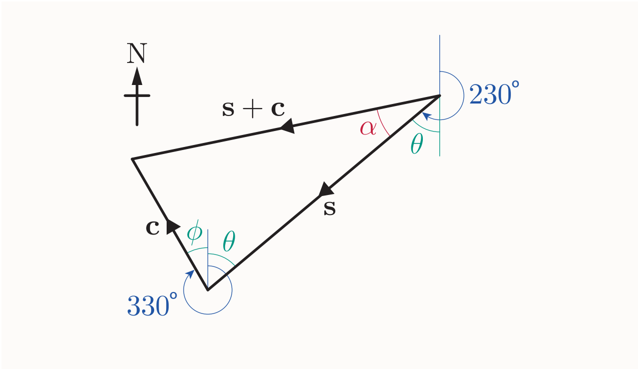
$\theta = \tan^{-1}\frac{2}{5} = 21.8\ldots°.$

The bearing of $\mathbf{s}+\mathbf{b}$ is $30° + 21.8\ldots° = 51.8\ldots°$.

So the resultant velocity of the boy is $10.8\,\mathrm{m\,s^{-1}}$ (to 1 d.p.) on a bearing of 52° (to the nearest degree).

Solution to Activity 33

Let $\mathbf{s}$ be the velocity of the ship in still water, and let $\mathbf{c}$ be the velocity of the current. The resultant velocity of the ship is $\mathbf{s}+\mathbf{c}$, as shown below.



We know that $|\mathbf{s}| = 5.7\,\mathrm{m\,s^{-1}}$ and $|\mathbf{c}| = 2.5\,\mathrm{m\,s^{-1}}$.

The angle marked θ at the tail of $\mathbf{s}$ is given by $\theta = 230° - 180° = 50°$. Since alternate angles are equal, the angle θ marked at the tip of $\mathbf{s}$ has the same size.

The angle ϕ marked at the tail of $\mathbf{c}$ is given by $\phi = 360° - 330° = 30°$.

So the bottom angle of the triangle is $\theta + \phi = 50° + 30° = 80°$.

Applying the cosine rule gives

$$|\mathbf{s}+\mathbf{c}|^2 = |\mathbf{s}|^2 + |\mathbf{c}|^2 - 2|\mathbf{s}||\mathbf{c}|\cos(\theta+\phi),$$

so

$$\begin{aligned}|\mathbf{s}+\mathbf{c}| &= \sqrt{5.7^2+2.5^2-2\times 5.7\times 2.5\times\cos 80°}\\ &= 5.813\ldots\,\mathrm{m\,s^{-1}}.\end{aligned}$$

The angle α can be found by using the sine rule:

$$\frac{|\mathbf{c}|}{\sin\alpha} = \frac{|\mathbf{s}+\mathbf{c}|}{\sin(\theta+\phi)}$$

$$\sin\alpha = \frac{|\mathbf{c}|\sin(\theta+\phi)}{|\mathbf{s}+\mathbf{c}|} = \frac{2.5\sin 80°}{5.813\ldots}.$$

Now,

$$\sin^{-1}\left(\frac{2.5\sin 80°}{5.813\ldots}\right) = 25.058\ldots°.$$

So $\alpha = 25.058\ldots°$ or $\alpha = 180° - 25.058\ldots° = 154.941\ldots°$.

But $|\mathbf{c}| < |\mathbf{s}+\mathbf{c}|$, so we expect $\alpha < \theta+\phi$; that is, $\alpha < 80°$. So $\alpha = 25.058\ldots°$, and hence the bearing of $\mathbf{s}+\mathbf{c}$ is $230° + 25.058\ldots° = 255.058\ldots°$.

The resultant velocity of the ship is $5.8\,\mathrm{m\,s^{-1}}$ (to 1 d.p.) on a bearing of 255° (to the nearest degree).