

# THE OPEN UNIVERSITY

## Analysing data (M248) Diagnostic Quiz

[Press the ↓ key to begin]

## 1. Introduction

In embarking on Analysing data (M248), basic mathematical competence is more important than any formal background in statistics. This quiz covers a number of key mathematical skills which are important for successful study of M248.

For M248, you need to have confidence with using mathematical notation and carrying out algebraic manipulation. You also need to be familiar with the notion of mathematical functions, including the logarithmic and the exponential functions. In addition, you will be required to use differentiation and integration: the module includes some revision material covering the specific differentiation and integration techniques used. You are assumed to have had some experience of basic use of a computer.

Try each question for yourself, using your calculator, then click on the green section letter (e.g. '(a)') to see the solution. Click on the  symbol at the end of the solution to return to the question. Use the  $\uparrow$  and  $\downarrow$  keys to move from section to section.

There is some advice on evaluating your performance at the end of the quiz.

## 2. Rounding and accuracy

### EXERCISE 1.

- (a) What is 0.4523 rounded to 1 decimal place?
- (b) What is 2.65847 rounded to 4 decimal places?

### EXERCISE 2.

- (a) What is 32.06165 rounded to 4 significant figures?
- (b) What is 24.9173 rounded to 1 significant figure?

### 3. Decimals and fractions

#### EXERCISE 3.

- (a) Evaluate  $2.8372 \times 1.8214$  to 3 decimal places.  
(b) Evaluate  $7.1946 \div 2.011$  to 3 significant figures.

#### EXERCISE 4.

- (a) Write down  $\frac{7}{8} \times \frac{2}{21}$  in its simplest fractional form.  
(b) Write down  $\frac{4}{10} \div \frac{16}{15}$  in its simplest fractional form.

## 4. Expressions, equations and functions

### EXERCISE 5.

- (a) Evaluate  $r\sqrt{\frac{n-2}{1-r^2}}$  for  $r = 0.6$  and  $n = 16$ , to 3 decimal places.
- (b) If  $1 - \frac{5}{x^2} = 0.95$ , find  $x$ .
- (c) If  $1 - e^{-3y} = 0.8$ , find  $y$  to 4 decimal places.

### EXERCISE 6.

- (a) Consider the function  $f(x) = 9 - x^2$ ,  $-3 \leq x \leq 3$ . Evaluate  $f(-1)$ ,  $f(2)$  and  $f(5)$ .
- (b) Consider the function  $g(x) = \frac{1}{(1-x)^2}$ . Evaluate  $g(-1)$ ,  $g(1)$  and  $g(5)$ .

## 5. Logarithms

**EXERCISE 7.** This exercise is about logarithms to base  $e$ , sometimes called ‘natural logarithms’ and denoted by ‘ $\ln$ ’, ‘ $\log_e$ ’ or just ‘ $\log$ ’.

- (a) Use your calculator to write down  $\log 3$  and  $\log 4$ , each to 4 decimal places.
- (b) Without using the logarithm key on your calculator again, can you calculate  $\log 12$  to 4 decimal places?
- (c) Is it true that logs of strictly positive\* real numbers must always be positive?

(\*Note that logs of numbers which are zero or negative are not defined. Can you see why?)

## 6. Powers

### EXERCISE 8.

- (a) Write  $3^3 \times 3^4$  in the form  $3^n$  and hence evaluate it.
- (b) Write  $(3^3)^4$  in the form  $3^n$  and hence evaluate it.
- (c) Write  $3^6/3^4$  in the form  $3^n$  and hence evaluate it.
- (d) Write  $3^5/3^5$  in the form  $3^n$  and hence evaluate it.

## 7. Summation notation

**EXERCISE 9.** Let  $x_1 = 2$ ,  $x_2 = 1$ ,  $x_3 = 3$ ,  $x_4 = -1$  and  $x_5 = 4$ . Let  $y_1 = 2$ ,  $y_2 = 0$ ,  $y_3 = 4$ ,  $y_4 = 5$  and  $y_5 = -2$ .

(a) Calculate  $\sum_{i=1}^5 x_i^2$ .

(b) Calculate  $\left(\sum_{i=1}^5 x_i\right)^2$ .

(c) Calculate  $\sum_{i=1}^5 x_i y_i$ .



## 8. Differentiation

**EXERCISE 10.** Find the derivatives of the following functions.

(a)  $f(x) = 3x^2$

(b)  $f(x) = \frac{2}{\sqrt{x}}$

(c)  $f(x) = 12 - 2x^2 + \frac{4}{x^2}$

(d)  $f(x) = 3e^{4x}$

(e)  $f(x) = (2x + 4)^5$

(f)  $f(x) = 4x^2(2 - x^2)$

## 9. Integration

EXERCISE 11. Find each of the following integrals.

(a)  $\int 4x^3 dx$

(b)  $\int (6x^2 + 3x + 5)dx$

(c)  $\int x^2(x + 1)^2 dx$

(d)  $\int 3(x^2 + 2x - 4)dx$

(e)  $\int_1^2 8x^2 dx$

(f)  $\int_1^2 x^2(4x - 2)dx$

## 10. Post-mortem

You should be familiar with the techniques covered by these Exercises before embarking on *Analysing data* (M248). If you were unsure of some of the detail in these areas, you could revise appropriate units from MST124 *Essential mathematics 1* (or its predecessor MST121 *Using mathematics*), if you have studied either of these modules previously.

Do contact your Student Support Team via StudentHome if you have any queries about your suitability for the module.

## Solutions to Exercises

**Exercise 1(a)** 0.4523 rounded to 1 decimal place is 0.5.



**Exercise 1(b)** 2.65847 rounded to 4 decimal places is 2.6585.



**Exercise 2(a)** 32.06165 rounded to 4 significant figures is 32.06.



**Exercise 2(b)** 24.9173 rounded to 1 significant figure is 20.  
(We are rounding to the nearest 10 in this case.)



**Exercise 3(a)**  $2.8372 \times 1.8214 = 5.16767608$ , which is 5.168 to 3 decimal places.





**Exercise 3(b)**  $7.1946 \div 2.011 = 3.577623073$  to the limits of calculator accuracy, which is 3.58 to 3 significant figures.



**Exercise 4(a)** Applying the usual cancellation rules, we have

$$\begin{aligned}\frac{7}{8} \times \frac{2}{21} &= \frac{1}{8} \times \frac{2}{3} \\ &= \frac{1}{4} \times \frac{1}{3} \\ &= \frac{1}{12}\end{aligned}$$

(after dividing above and below by 7, then by 2).



**Exercise 4(b)** Remembering that  $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ , where  $a, b, c, d$  represent any non-zero real numbers, and applying the cancellation rules, we get

$$\begin{aligned}\frac{4}{10} \div \frac{16}{15} &= \frac{4}{10} \times \frac{15}{16} \\ &= \frac{4}{2} \times \frac{3}{16} \\ &= \frac{1}{2} \times \frac{3}{4} \\ &= \frac{3}{8}.\end{aligned}$$



**Exercise 5(a)** Substituting for  $r$  and  $n$ , we get

$$\begin{aligned}r\sqrt{\frac{n-2}{1-r^2}} &= 0.6 \times \sqrt{\frac{16-2}{1-0.6^2}} \\ &= 0.6 \times \sqrt{\frac{14}{1-0.36}} \\ &= 0.6 \times \sqrt{\frac{14}{0.64}} \\ &= 0.6 \times \sqrt{21.875} \\ &= 0.6 \times 4.677071733 \\ &= 2.80624304 \\ &= 2.806\end{aligned}$$

to 3 decimal places.



**Exercise 5(b)** Remember that you can add or subtract the same quantities to or from both sides. So

$$1 - \frac{5}{x^2} = 0.95$$

becomes

$$\frac{5}{x^2} = 1 - 0.95 = 0.05.$$

Inverting both sides gives

$$\frac{x^2}{5} = \frac{1}{0.05} = 20,$$

that is,  $x^2 = 100$ , or  $x = \pm 10$ .



**Exercise 5(c)** Rearranging as before,

$$1 - e^{-3y} = 0.8$$

becomes

$$e^{-3y} = 1 - 0.8 = 0.2.$$

Recalling that  $a^{-b} = \frac{1}{a^b}$ , for any real numbers  $a$  and  $b$ ,  $a \neq 0$ ,

$$\frac{1}{e^{3y}} = 0.2$$

and so

$$e^{3y} = \frac{1}{0.2} = 5.0.$$

Finally, recalling that the logarithm function is the inverse of the exponential function,

$$3y = \log_e 5.0 = 1.609437912,$$

and so

$$y = 0.5364793041 = 0.5365,$$

to 4 decimal places.



**Exercise 6(a)** For the function  $f(x) = 9 - x^2$ ,  $-3 \leq x \leq 3$ , putting  $x = -1$  yields

$$\begin{aligned} f(-1) &= 9 - (-1)^2 \\ &= 9 - 1 \\ &= 8. \end{aligned}$$

Putting  $x = 2$  yields

$$\begin{aligned} f(2) &= 9 - 2^2 \\ &= 9 - 4 \\ &= 5. \end{aligned}$$

The value  $x = 5$  lies outside the range of values of  $x$  on which the function  $f$  is defined. We cannot therefore evaluate  $f(5)$ .



**Exercise 6(b)** For the function  $g(x) = 1/(1 - x)^2$ , putting  $x = -1$  yields

$$\begin{aligned}g(-1) &= \frac{1}{(1 - (-1))^2} \\ &= \frac{1}{2^2} \\ &= \frac{1}{4}.\end{aligned}$$

The function  $g$  is not defined for  $x = 1$ . Since  $1 - 1 = 0$  and so the denominator is then 0 (and we cannot divide by 0).

Finally putting  $x = 5$  yields

$$\begin{aligned}g(5) &= \frac{1}{(1 - 5)^2} \\ &= \frac{1}{(-4)^2} \\ &= \frac{1}{16}.\end{aligned}$$





**Exercise 7(a)** To the limits of calculator accuracy,

$$\log 3 = 1.098612289$$

and is therefore 1.0986 to 4 decimal places.

(If you get something different, check that you have used the right key on your calculator. In particular, if you have used the key which provides logarithms to base 10, you would get 0.4771, to 4 decimal places.)

Similarly,  $\log 4 = 1.386294361 = 1.3863$ , to 4 decimal places.

(Again, using the 'base 10' key would give 0.6021, to 4 decimal places.)



**Exercise 7(b)** Using the rule  $\log ab = \log a + \log b$  yields

$$\begin{aligned}\log 12 &= \log(3 \times 4) \\ &= \log 3 + \log 4 \\ &= 1.098612289 + 1.386294361 \\ &= 2.48490665 \\ &= 2.4849,\end{aligned}$$

to 4 decimal places.



**Exercise 7(c)** No. Logarithms of positive numbers strictly less than 1 are negative. Check this on your calculator. (Can you see why this should be the case?)



**Exercise 8(a)** Using the rule  $x^a \times x^b = x^{a+b}$  gives

$$\begin{aligned}3^3 \times 3^4 &= 3^{3+4} \\ &= 3^7 \\ &= 2187.\end{aligned}$$



**Exercise 8(b)** Using the rule  $(x^a)^b = x^{ab}$  gives

$$\begin{aligned}(3^3)^4 &= 3^{3 \times 4} \\ &= 3^{12} \\ &= 531441.\end{aligned}$$



**Exercise 8(c)** Using the rule  $x^a/x^b = x^{a-b}$  gives

$$\begin{aligned} 3^6/3^4 &= 3^{6-4} \\ &= 3^2 \\ &= 9. \end{aligned}$$



**Exercise 8(d)** Here, we have

$$\begin{aligned}3^5/3^5 &= 3^{5-5} \\ &= 3^0 \\ &= 1,\end{aligned}$$

noting that, for any real number  $x$ ,  $x^0 = 1$ .



**Exercise 9(a)**

$$\begin{aligned}\sum_{i=1}^5 x_i^2 &= 2^2 + 1^2 + 3^2 + (-1)^2 + 4^2 \\ &= 4 + 1 + 9 + 1 + 16 \\ &= 31.\end{aligned}$$





**Exercise 9(b)**

$$\begin{aligned}\left(\sum_{i=1}^5 x_i\right)^2 &= (2 + 1 + 3 + (-1) + 4)^2 \\ &= 9^2 \\ &= 81.\end{aligned}$$



**Exercise 9(c)**

$$\begin{aligned}\sum_{i=1}^5 x_i y_i &= (2 \times 2) + (1 \times 0) + (3 \times 4) + ((-1) \times 5) + (4 \times (-2)) \\ &= 4 + 0 + 12 - 5 - 8 \\ &= 3.\end{aligned}$$



**Exercise 10(a)** Use the rule that if  $f(x) = ax^k$ , then  $f'(x) = kax^{k-1}$ . So, when

$$f(x) = 3x^2,$$

then

$$f'(x) = 2 \times 3x^{2-1} = 6x.$$



**Exercise 10(b)** Use the rule that if  $f(x) = ax^k$ , then  $f'(x) = kax^{k-1}$ . So, when

$$f(x) = \frac{2}{\sqrt{x}} = 2x^{-1/2},$$

then

$$\begin{aligned} f'(x) &= -\frac{1}{2} \times 2x^{-1/2-1} \\ &= -1x^{-3/2} \\ &= \frac{-1}{\sqrt{x^3}}. \end{aligned}$$



**Exercise 10(c)** Use the rule that if

$f(x) = g(x) + h(x) + \cdots + \ell(x)$ , where  $g(x), h(x), \dots, \ell(x)$  are any functions of  $x$ , then

$$f'(x) = g'(x) + h'(x) + \cdots + \ell'(x).$$

So, when

$$f(x) = 12 - 2x^2 + \frac{4}{x^2} = 12 - 2x^2 + 4x^{-2},$$

then

$$\begin{aligned} f'(x) &= 0 - 2 \times 2x^{2-1} + (-2) \times 4x^{-2-1} \\ &= -4x - 8x^{-3} \\ &= -4x - \frac{8}{x^3}. \end{aligned}$$



**Exercise 10(d)** Use the rule that if  $f(x) = ae^{kx}$ , then  $f'(x) = kae^{kx}$ . So, when

$$f(x) = 3e^{4x},$$

then

$$f'(x) = 4 \times 3e^{4x} = 12e^{4x}.$$



**Exercise 10(e)** By the chain rule, the derivative of  $f(x) = h(g(x))$  is

$$f'(x) = g'(x)h'(g(x)).$$

So, when

$$f(x) = (2x + 4)^5,$$

let

$$h(y) = y^5, \quad y = g(x) = 2x + 4.$$

Then

$$h'(y) = 5y^4 \text{ and } g'(x) = 2.$$

It follows that

$$\begin{aligned} f'(x) &= 2 \times 5y^4 \\ &= 10(2x + 4)^4. \end{aligned}$$



**Exercise 10(f)** Use the rule that if  $f(x) = g(x) \times h(x)$ , where  $g(x)$  and  $h(x)$  are any functions of  $x$ , then

$$f'(x) = g'(x)h(x) + g(x)h'(x).$$

So, when

$$f(x) = 4x^2(2 - x^2),$$

let

$$g(x) = 4x^2, \quad h(x) = 2 - x^2,$$

so that

$$g'(x) = 8x, \quad h'(x) = -2x.$$

Then

$$\begin{aligned} f'(x) &= 8x \times (2 - x^2) + 4x^2 \times (-2x) \\ &= 8x(2 - x^2) - 8x^3 \\ &= 8x(2 - x^2 - x^2) \\ &= 16x(1 - x^2) \end{aligned}$$





**Exercise 11(a)** Use the rule that for  $k \neq -1$ ,

$$\int ax^k dx = \frac{ax^{k+1}}{k+1} + c.$$

So

$$\begin{aligned}\int 4x^3 dx &= \frac{4x^4}{4} + c \\ &= x^4 + c\end{aligned}$$



**Exercise 11(b)** Use the rule that if  $g(x)$ ,  $h(x)$  and  $\ell(x)$  are any functions of  $x$ ,

$$\begin{aligned} & \int \{g(x) + h(x) + \cdots + \ell(x)\} dx \\ &= \int g(x)dx + \int h(x)dx + \cdots + \int \ell(x)dx. \end{aligned}$$

So

$$\begin{aligned} \int (6x^2 + 3x + 5)dx &= \frac{6x^3}{3} + \frac{3x^2}{2} + 5x + c \\ &= 2x^3 + \frac{3x^2}{2} + 5x + c. \end{aligned}$$



**Exercise 11(c)** The brackets need expanding before the function can be integrated. So

$$\begin{aligned}\int x^2(x+1)^2 dx &= \int x^2(x^2 + 2x + 1) dx \\ &= \int (x^4 + 2x^3 + x^2) dx \\ &= \frac{x^5}{5} + \frac{2x^4}{4} + \frac{x^3}{3} + c \\ &= \frac{x^5}{5} + \frac{x^4}{2} + \frac{x^3}{3} + c.\end{aligned}$$



**Exercise 11(d)** Use the rule that

$$\int af(x)dx = a \int f(x)dx.$$

So

$$\begin{aligned}\int 3(x^2 + 2x - 4)dx &= 3 \int (x^2 + 2x - 4)dx \\ &= 3 \left( \frac{x^3}{3} + \frac{2x^2}{2} - 4x \right) + c \\ &= x^3 + 3x^2 - 12x + c.\end{aligned}$$



**Exercise 11(e)**

$$\begin{aligned}\int_1^2 8x^2 dx &= \left[ \frac{8x^3}{3} \right]_1^2 \\ &= \frac{8 \times 2^3}{3} - \frac{8 \times 1^3}{3} \\ &= \frac{64}{3} - \frac{8}{3} \\ &= \frac{56}{3}\end{aligned}$$



**Exercise 11(f)**

$$\begin{aligned}\int_1^2 x^2(4x - 2)dx &= \int_1^2 (4x^3 - 2x^2)dx \\ &= \left[ \frac{4x^4}{4} - \frac{2x^3}{3} \right]_1^2 \\ &= \left[ x^4 - \frac{2x^3}{3} \right]_1^2 \\ &= 2^4 - \frac{2 \times 2^3}{3} - \left( 1^4 - \frac{2 \times 1^3}{3} \right) \\ &= 16 - \frac{16}{3} - 1 + \frac{2}{3} \\ &= 10\frac{1}{3}.\end{aligned}$$

