The Open University
Mathematics, Computing and Technology

MU123 Discovering mathematics

Unit 5 Algebra

Prepared by the Course Team

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Introduction

In Unit 2 you met the idea of using letters to represent numbers. In this unit you'll learn much more about this sort of mathematics, which is called **algebra**. You'll begin to see how useful it can be, and you'll begin to learn the techniques that will allow you to make the most of it.

You'll need to use the algebraic skills covered in this unit when you study some of the later units in the course, and they're essential for any further mathematics courses. So it's important that you become proficient in them. The only way to do that is to practise them! You'll have lots of opportunity for that in this unit.

The word 'algebra' is derived from the title of the treatise *Al-kitab-al mukhtasar fi hisab al-jabr* (Compendium on calculation by completion and reduction), written by the Central Asian mathematician Muḥammad ibn Mūsā al-Khwārizmī in around 825 CE. This treatise deals with solving linear and quadratic equations, which you'll learn about in this course, starting with linear equations in this unit. The treatise doesn't use algebra in the modern sense, as no letters or other symbols are used to represent numbers. Modern algebra developed gradually over time, and it was not until the sixteenth and seventeenth centuries that it emerged in forms we recognise and use today.

Muḥammad ibn Mūsā al-Khwārizmī was a member of the House of Wisdom in Baghdad, a research centre established by the Caliph al-Ma'mūn. His name is the origin of the word 'algorithm'. As you saw in Unit 2, an algorithm is a procedure for solving a problem or doing a calculation.

1 Why learn algebra?

What's the point of learning algebra? Why is it useful? In this section you'll see some answers to these questions.

1.1 Proving mathematical facts

This first activity is about a number trick.

Activity 1 *Think of a number*

Try the following number trick.

Think of a number.
Double it.
Add 7.
Double the result.
Add 6.
Divide by 4.
Take away the number you first thought of.
Find the corresponding letter of the alphabet.
Name an animal beginning with that letter.

Choose a fairly small number, so that the arithmetic is easy!

For example, if your number is 3, then your letter is C, the third letter of the alphabet.

Now look at the solution on page 48.

Activity 2 *Think of another number*

Try the trick in Activity 1 again, choosing a different number to start with. Check your solution and then read the discussion below.

In Activities 1 and 2 you probably found that with both your starting numbers you obtained the number 5 in the third-last step (the last step involving a mathematical calculation) and so each time you obtained the letter E. If you didn't, then check your arithmetic! When asked to name an animal beginning with the letter E, nearly everyone thinks of 'elephant'.

You might like to try the trick on a friend, or a child who's old enough to do the arithmetic. If you do, remember to write the word 'elephant' on a piece of paper beforehand, ready to reveal at the end of the trick.

The idea behind the trick is that the number 5 is obtained in the last mathematical step, no matter what the starting number is. But how can you be sure that the trick works for *every possible* starting number? You can't test them all individually as there are infinitely many possibilities.

There's a way to check this – using algebra. In Sections 2 to 4 you'll learn the algebraic techniques that are needed, and you'll see how to use them to check that the trick always works.

As you saw earlier in the course, a demonstration that a piece of mathematics *always* works is called a **proof**. Proofs of mathematical facts are needed in all sorts of contexts, and algebra is usually the way to construct them.

1.2 Finding and simplifying formulas

Suppose that a baker makes a particular type of loaf. Each loaf costs 69p to make, and is sold for £1.24. The baker sells all the loaves that he makes.

On a particular day, the baker makes 30 loaves. Let's calculate the profit that he makes from them. The total cost, in £, of making the loaves is

$$30 \times 0.69 = 20.70.$$

The total amount of money, in £, paid for the loaves by customers is

$$30 \times 1.24 = 37.20.$$

So the profit in £ is given by

$$37.20 - 20.70 = 16.50.$$

That is, the profit is £16.50.

On a different day, the baker might make a different number of loaves. It would be useful for him to have a formula to help him calculate the profit made from *any* number of loaves. To obtain the formula, we represent the number of loaves by a letter, say n, and work through the same calculation as above, but using n in place of 30.

The total cost, in £, of making the loaves is

$$n \times 0.69 = 0.69n.$$

Remember that if a letter and a number are multiplied together, then we omit the multiplication sign, and we write the number first. So we write

$$n \times 0.69$$

as

$$0.69n.$$

The total amount of money, in £, paid for the loaves by customers is

$$n \times 1.24 = 1.24n.$$

So if we represent the profit by £P, then we have the formula

$$P = 1.24n - 0.69n.$$

Activity 3 *Using a formula*

Use the formula above to calculate the profit for 48 loaves.

The formula makes it easy to calculate the profit, because you don't need to think through the details of the calculation. You just substitute in the number and do a numerical calculation. This is the advantage of using a formula.

The task of calculating the profit can be made even more straightforward. It's possible to find a *simpler* formula for P, by looking at the situation in a different way. The profit, in £, for each loaf of bread is

$$1.24 - 0.69 = 0.55.$$

So the profit, in £, for n loaves of bread is

$$n \times 0.55 = 0.55n.$$

So we have the alternative formula

$$P = 0.55n.$$

This formula, like the first one, can be used for any value of n.

Activity 4 *Using a better formula*

Use the new formula above to find the profit for 48 loaves of bread.

The alternative formula for the profit is better because it is simpler and using it involves less calculation.

Usually it's not easy to find the simplest formula for a particular purpose by thinking about the situation in different ways. Instead, it's best to find whatever formula you can, and use algebra to turn it into a simpler form. You'll begin to learn how to do this later in the unit.

Algebra can also help you to *find* formulas. The formula for the baker's profit was obtained directly from the situation that it describes, but it's often easier to obtain formulas by using other formulas that you know already. Algebra is needed for this process, and it's also needed to turn the new formula into a simpler form. You'll find out more about this in Unit 7.

1.3 Answering mathematical questions

Consider the following.

> A school has stated that 30% of the children who applied for places at the school were successful. It allocated 150 places. How many children applied?

To help you to think clearly about questions like this, it helps to represent the number that you want to find by a letter. Let's use N to represent the number of children who applied.

The next step is to write down what you know about the letter in mathematical notation. We know that 30% of N is 150. That is,

$$\frac{30}{100} \times N = 150,$$

which can be written more concisely as

$$\tfrac{3}{10}N = 150. \qquad (1)$$

This is an example of an *equation*. To answer the question about the school places, you have to find the value of N that makes the equation correct when it's substituted in. This is called *solving* the equation.

You'll learn exactly what 'equation' means in the next section.

One way to solve the equation is as follows:

> *Three*-tenths of N is 150,
> so *one*-tenth of N is $150 \div 3 = 50$,
> so N is $10 \times 50 = 500$.

You'll learn a different way to solve this equation in Section 5.

So the number of children who applied was 500.

You can confirm that this is the right answer by checking that equation (1) is correct when $N = 500$ is substituted in.

Activity 5 *Using an equation*

Two-fifths of the toddlers in a village attend the local playscheme. Twenty-four toddlers attend the playscheme.

(a) Let the total number of toddlers in the village be T. Write down an equation (similar to equation (1)) involving T.

(b) Find the value of T.

You could probably have answered the questions about the school places and the toddlers without using equations. But now read the following.

> Catherine wants to contribute to a charitable cause, using her credit card and a donations website. The donations company states that from each donation, first it will deduct a 2% charge for credit card use, then it will deduct a charge of £3 for use of its website, and then the remaining money will be increased by 22% due to tax payback. How much money (to the nearest penny) must Catherine pay if she wants the cause to receive £40?

This question is a bit harder! But it can be answered by writing the information in the question as an equation and using algebra, as you'll see later in the unit.

The advantage of writing the information in a question as an equation is that it reduces the problem of answering the question to the problem of solving an equation. The equation may be more complicated than the two you've seen in this section, but there are standard algebraic techniques for solving many equations, even complicated ones. You'll learn some of these techniques in this unit.

In this section you've seen just the beginnings of what algebra can do. Algebra is used in many different fields, including science, computer programming, medicine and finance. For example, it's used to create formulas so that computer programs can carry out many different tasks, from calculating utility bills to producing images on screens. And it's used in mathematical models, so that predictions, such as those about the economy and climate change, can be made by solving equations. Algebra allows us to describe, analyse and understand the world, to solve problems and to set up procedures. Our lives would be very different if algebra had not been invented!

You'll learn more about the power of algebra if you take further courses in mathematics.

1.4 How to learn algebra

Now that you've seen some reasons why algebra is useful, you should be ready to learn more about it.

Algebraic notation is 'the language of mathematics', and it takes time to learn, like any new language. So don't worry if you don't absorb some of the ideas immediately. Allow yourself time to get to grips with them, and keep practising the techniques. You learn a language by using it, not by reading about it. Remember that any difficulties will often be quickly sorted out if you call your tutor or post a question on the online forum.

The activities in this unit have been designed to teach you algebra in a step-by-step manner. They give you the opportunity to practise, and become familiar with, each new technique before you meet the next one – this is important, because most of the techniques build on techniques introduced previously. You should aim to do *all* the activities, and you should do them in the order in which they're presented.

Do the activities even if they look easy – many students find that there are small gaps or misunderstandings in their skills that they're unaware of until they attempt the activities or check their answers against the correct answers. This unit gives you the opportunity to identify and deal with such problems, and so prevent them causing difficulties later. You may even find that some activities throw new light on techniques with which you're familiar. Do *all* the parts of each activity – the parts are often different in subtle ways, and frequently the later parts are more challenging than the earlier ones.

Many activities are preceded by worked examples, which demonstrate the techniques needed. Before you attempt each activity, read through any relevant worked examples, or watch the associated tutorial clips if they're available, and try to make sure that you understand each step.

Tutorial clips are available for the more complex techniques. You will probably find that you learn more effectively by watching the tutorial clips rather than by just reading through the worked examples.

Set out your solutions in a similar way to those in the worked examples. Remember that any green text within the think bubbles icons isn't part of the solutions, but any other words in the solutions *are* part of them, and similar explanations should be included in your own solutions.

Enjoy learning the new skills!

2 Expressions

In this section you'll learn some terminology used in algebra and a useful technique – collecting like terms.

2.1 What is an expression?

In the course you've worked with various formulas, such as

$$P = 0.55n \quad \text{and} \quad T = \frac{D}{5} + \frac{H}{600}.$$

The first of these is the 'baker's profit' formula from Subsection 1.2, and the second is Naismith's Rule from Unit 2.

These formulas involve

$$P, \quad 0.55n, \quad T \quad \text{and} \quad \frac{D}{5} + \frac{H}{600},$$

which are all examples of *algebraic expressions*. An **algebraic expression**, or just **expression** for short, is a collection of letters, numbers and/or mathematical symbols (such as $+$, $-$, $\times$, $\div$, brackets, and so on), arranged in such a way that if numbers are substituted for the letters, then you can work out the value of the expression.

So, for example, $5n + 20$ is an expression, but $5n + \div 20$ is not an expression, because '$+\div$' doesn't make sense.

To make expressions easier to work with, we write them concisely in the ways you saw earlier in the course. In particular, we usually omit multiplication signs; things that are multiplied are just written next to each other instead. But it's sometimes helpful to include some multiplication signs in an expression – and a multiplication sign between two numbers can't be omitted.

You saw how to write formulas concisely in Unit 2, Subsection 3.2.

For example, $0.55 \times n$ is written as $0.55n$.

When you're working with expressions, the following is the key thing to remember.

> Letters represent numbers, so the normal rules of arithmetic apply to them in exactly the same way as they apply to numbers.

In particular, the BIDMAS rules apply to the letters in expressions.

The BIDMAS rules were covered in Unit 1, Subsection 2.1 and are summarised in the Handbook.

When you substitute numbers for the letters in an expression and work out its value, you're **evaluating** the expression.

Example 1 *Evaluating an expression*

Evaluate the expression

$$4x^2 - 5y$$

when $x = 2$ and $y = -3$.

This expression, like many other expressions in this unit, has no special meaning.

Solution

If $x = 2$ and $y = -3$, then

$$\begin{aligned} 4x^2 - 5y &= 4 \times 2^2 - 5 \times (-3) \\ &= 4 \times 4 - (-15) \\ &= 16 + 15 \\ &= 31. \end{aligned}$$

Remember to apply the BIDMAS rules. Be particularly careful in part (d).

Activity 6 *Evaluating expressions*

Evaluate the following expressions when $a = -2$ and $b = 5$.

(a) $\frac{5}{2} + a$ (b) $-a + ab$ (c) ab^2 (d) $b + 3(b - a)$

Expressions don't have to contain letters – for example, $(2 + 3) \times 4$ is an expression.

Every expression can be written in many different ways. For example, multiplication signs can be included or omitted. As another example, the expression $x + x$ can also be written as $2x$. That's because adding a number to itself is the same as multiplying it by 2.

For instance, $3 + 3$ means the same as 2×3.

If two expressions are really just the same, but written differently, then we say that they're different **forms** of the same expression, or that they're **equivalent** to each other. We indicate this by writing an equals sign between them. For example, because $x + x$ is equivalent to $2x$, we write

$$x + x = 2x.$$

If two expressions are equivalent, then, whatever values you choose for their letters, the two expressions have the same value as each other.

Activity 7 *Checking whether expressions are equivalent*

Which of the following statements are correct?

(a) $u + u + u = 3u$ (b) $a^2 \times a = a^3$ (c) $2a \div 2 = a$

(d) $p^2 \times p^3 = p^6$ (e) $z + 2z = 3z$ (f) $6c \div 2 = 3c$

(g) $a - b - 2c = a + (-b) + (-2c)$ (h) $3n \div n = n$

You saw another example of equivalent expressions in Subsection 1.2, when two different formulas, $P = 1.24n - 0.69n$ and $P = 0.55n$, were found for the baker's profit. Since the formulas must give the same values,

$$1.24n - 0.69n = 0.55n.$$

When we write an expression in a different way, we say that we're **rearranging**, **manipulating** or **rewriting** the expression. Often the aim of doing this is to make the expression simpler, as with the formula for the baker's profit. In this case we say that we're **simplifying** the expression.

The first known use of an equals sign was by the Welsh mathematician Robert Recorde, in his algebra textbook *The whetstone of witte*, published in 1557. He justified the use of two parallel line segments to indicate equality as follows: *bicause noe 2 thynges can be moare equalle.*

We use equals signswhen we're working with expressions, but expressions don't *contain* equals signs. For example, the statements

$$x + x = 2x \quad \text{and} \quad 1.24n - 0.69n = 0.55n$$

aren't expressions – they're equations. An **equation** is made up of *two* expressions, with an equals sign between them.

There's a difference between the two equations above and the ones that arose from the questions about school places and toddlers in Subsection 1.3. In that subsection, the number of children who applied to a school was found using the equation

$$\tfrac{3}{10}N = 150.$$

This equation is correct for *only one* value of N (it turned out to be 500). In contrast, the equations

$$x + x = 2x \quad \text{and} \quad 1.24n - 0.69n = 0.55n$$

are correct for *every* value of x and n, respectively. Equations like these, which are true for all values of the variables, are called **identities**. The different types of equation don't usually cause confusion in practice, as you know from the context which type you're dealing with.

There's more about identities in Unit 9.

There are similar differences in the use of letters to represent numbers. When the equation $\frac{3}{10}N = 150$ was used in Subsection 1.3, the letter N represented a *particular* number – it was just that we didn't know what that number was. This type of letter is called an **unknown**. In contrast, in the equation $x + x = 2x$ above, the letter x represents *any* number. A letter that represents any number (or any number of a particular type, such as any integer) is called a **variable**, as you saw in Unit 2. Usually you don't need to think about whether a letter is an unknown or a variable. Both types represent numbers, so the same rules of arithmetic apply in each case.

The first person to represent known quantities by consonants and unknowns by vowels (so that numbers were replaced throughout by letters) was the French mathematician François Viète (1540–1603). But he had no systematic way of denoting powers and the word *aequatur* for equality was retained. His treatise *In artem analyticem isagoge* (Introduction to the analytic art) of 1591 gives methods for solving equations, including ones more complicated than those in this course.

Viète also wrote books on astronomy, geometry and trigonometry, but he was never employed as a professional mathematician. He was trained in law, and followed a legal career for a few years before leaving the profession to oversee the education of the daughter of a local aristocratic family. His later career was spent in high public office, apart from a period of five years when he was banished from the court in Paris for political and religious reasons. Throughout his life, the only time he could devote to mathematics was when he was free from official duties.

Figure 1 François Viète

2.2 What is a term?

Some expressions are lists of things that are all added or subtracted. Here's an example:

$$-2xy + 3z - y^2.$$

In an expression of this sort, the things that are added or subtracted are called the **terms**. The terms of the expression above are

$$-2xy, \quad +3z \quad \text{and} \quad -y^2.$$

The plus or minus sign at the start of each term is *part of the term*. While you're getting used to working with terms, it can be helpful to mark them like this:

$$\underbrace{-2xy}\,\underbrace{+\,3z}\,\underbrace{-\,y^2}.$$

A sign *after* a term is part of the next term.

You need to make sure that the sign *at the start* of each term is included along with the rest of the term.

If the first term of an expression has no sign, then the term is added to the other terms, so really it has a plus sign – it's just that we normally don't write a plus sign in front of the first term of an expression. For example, if you have the expression

$$4a + c - 7\sqrt{b} - 5,$$

then you could write a plus sign at the start and mark the terms like this:

$$\underbrace{+4a}\ \underbrace{+c}\ \underbrace{-7\sqrt{b}}\ \underbrace{-5}\,.$$

Its terms are

$$+4a, \quad +c, \quad -7\sqrt{b} \quad \text{and} \quad -5.$$

Activity 8 *Identifying terms of expressions*

For each expression below, copy the expression, mark the terms and write down a list of the terms.

(a) $x^3 - x^2 + x + 1$ (b) $2mn - 3r$ (c) $-20p^2q^2 + \frac{1}{4}p - 18 - \frac{1}{3}q$

Remember that when you handwrite a lower-case x in mathematics, you should make it look different from a multiplication sign. One way to do this is to write it as a 'backwards c' followed by a 'normal c', like this:

When we discuss the terms of an expression, we often omit the plus signs. This is convenient in the same way that it's convenient to write the number +3 as 3. So, for example, we might say that the expression

$$-2xy + 3z - y^2 \tag{2}$$

has terms

$$-2xy, \quad 3z \quad \text{and} \quad -y^2.$$

We *never* omit the minus signs! And, of course, we *never* omit the plus sign of a term when writing the term as part of an expression, unless it's the first term.

There's a useful way to think of the relationship between an expression and its terms.

Remember that a *sum* of numbers is what you get by adding them. For example, the sum of 1, 4 and 7 is $1 + 4 + 7 = 12$.

> An expression is equivalent to the sum of its terms.

For example, here is expression (2) written as the sum of its terms:

$$-2xy + 3z - y^2 = -2xy + 3z + (-y^2).$$

The expression on the right is obtained by adding the terms of the expression on the left. The two expressions are equivalent because subtracting y^2 is the same as adding the negative of y^2.

You saw in Unit 1 that subtracting a number is the same as adding its negative. For example, $1 - 3$ is the same as $1 + (-3)$.

You saw another example of an expression written as the sum of its terms in Activity 7(g):

$$a - b - 2c = a + (-b) + (-2c).$$

Because the order in which numbers are added doesn't matter, you can change the order of the terms in an expression however you like, and obtain an equivalent expression, as long as you keep each term together with its sign. For example, you can swap the order of the first two terms in the expression

$$-2xy + 3z - y^2$$

to give

$$3z - 2xy - y^2.$$

This expression could be written as

$$+3z - 2xy - y^2,$$

but we usually omit a plus sign in front of a first term.

Or you can reverse the original order of the terms to give

$$-y^2 + 3z - 2xy.$$

All three of these expressions are equivalent to each other.

When you do the next activity, you'll probably find it helpful to begin by marking the terms in the way shown on page 10, including their signs, of course. Then think of moving the marked terms around.

Remember that you may need to write a plus sign in front of the first term.

Activity 9 *Changing the order of terms*

Write each of the following expressions with its terms in reverse order.

(a) $-X + 20Y - 5Z$ (b) $2u - 3uv$ (c) $4i - j + 5$

(d) $a - b + c + d$

Changing the order of the terms doesn't simplify an expression, but some methods for simplifying expressions are easier to apply if you rearrange the terms first.

A term in an expression may be just a number, like 4, $\frac{1}{2}$ or -5. If so, we say that it's a **constant term**, or just a **constant**. For example, the expression

$$3pq - 2 + 5p^2$$

has one constant term, -2.

It's called a constant term because, unlike other terms, its value doesn't change when the values of the letters in the expression are changed.

On the other hand, if a term is of the form

a number × a combination of letters,

then the number is called the **coefficient** of the term, and we say that the term is a term *in* whatever the combination of letters is. For example,

$2xy$ has coefficient 2 and is a term in xy;
$-3z$ has coefficient -3 and is a term in z;
$\frac{2}{3}c^2$ has coefficient $\frac{2}{3}$ and is a term in c^2.

The word 'coefficient' was introduced by François Viète (see page 9).

You may be tempted to think that terms like a and $-a$ don't have coefficients. However, because they are equivalent to $1a$ and $-1a$, respectively, they have coefficients 1 and -1, respectively.

We normally write a rather than $1a$, and $-a$ rather than $-1a$, for conciseness.

Here are some more examples:

y^3 has coefficient 1 and is a term in y^3;
$-ab^2c$ has coefficient -1 and is a term in ab^2c.

Activity 10 *Identifying coefficients*

Write down the coefficient of:

(a) the third term in $2x^2 + 3xy + 4y^2$

(b) the second term in $2\sqrt{p} - 9\sqrt{q} - 7$

(c) the third term in $2x + 5 + x^2$

(d) the first term in $-a^2b + 2c$

(e) the term in m^2 in $1 + 2m - 3m^2$

(f) the term in b in $ab + 2b + b^2$.

Activity 11 *Indentifying constant terms*

For each of parts (a)–(f) in Activity 10 above, write down any constant terms in the expression.

2.3 Collecting like terms

In this subsection you'll learn the first of several techniques for simplifying expressions: *collecting like terms.*

Let's look first at how it works with numbers. If you have 2 batches of 4 dots, and another 3 batches of 4 dots, then altogether you have

2 + 3 batches of 4 dots, that is, 5 batches of 4 dots.

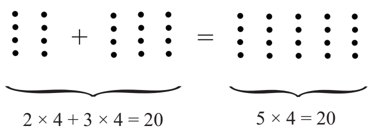

Figure 2

This is shown in Figure 2, and you can express it by writing

$$2 \times 4 + 3 \times 4 = 5 \times 4.$$

Of course, this doesn't work just with batches of four dots. For example, in Figure 3 we have

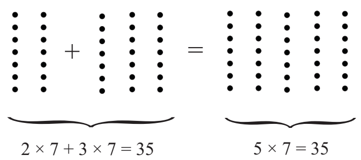

Figure 3

$$2 \times 7 + 3 \times 7 = 5 \times 7.$$

In fact, no matter what number a is,

$$2a + 3a = 5a.$$

This gives you a way of simplifying expressions that contain a number of batches of something, added to another number of batches *of the same thing*. For example,

$$15ab + 14ab = 29ab,$$
$$4\sqrt{A} + 5\sqrt{A} = 9\sqrt{A},$$
$$5x^2 + 25x^2 = 30x^2.$$

But you can't simplify the following expressions in this way, because they don't contain batches *of the same thing*:

$$3a + 5ab,$$
$$4X + 9X^2.$$

Terms that are 'batches of the same thing' are called *like* terms. That is, **like terms** are terms that are the same except possibly for the coefficients. Like terms can always be collected in a similar way to the examples above: you just combine the coefficients.

You can add any number of like terms, and you can also subtract them. The coefficients don't have to be whole numbers: they can be fractions, decimal numbers, negative numbers – any numbers at all.

Example 2 *Collecting like terms*

Simplify the following expressions.

(a) $12m + 15m - 26m$ (b) $0.5XY^2 + 0.1XY^2$ (c) $5p - p$

(d) $\frac{1}{3}d - 2d$

Solution

(a) $12m + 15m - 26m = (12 + 15 - 26)m = 1m = m$

(b) $0.5XY^2 + 0.1XY^2 = (0.5 + 0.1)XY^2 = 0.6XY^2$

(c) $5p - p = 5p - 1p = (5 - 1)p = 4p$

(d) $\frac{1}{3}d - 2d = \frac{1}{3}d - \frac{6}{3}d = (\frac{1}{3} - \frac{6}{3})d = -\frac{5}{3}d$

Notice that in the solution to Example 2(d), the fractional coefficients were not converted to approximate decimal values. In algebra you should work with *exact* numbers, such as $\frac{1}{3}$ and $\sqrt{5}$, rather than decimal approximations, wherever possible. However, if you're using algebra to solve a practical problem, then you may have to use decimal approximations.

Activity 12 *Collecting like terms*

Simplify the following expressions.

(a) $8A + 7A$ (b) $-5d + 8d - 2d$ (c) $-7z + z$

(d) $1.4pq + 0.7pq - pq$ (e) $\frac{1}{2}n^2 - \frac{1}{3}n^2$

It's easier to spot like terms if you make sure that all the letters in each term are written in alphabetical order. For example, $5st$ and $2ts$ are like terms – this is easier to see if you write the second one as $2st$.

You can always change the order in which things are multiplied, as this doesn't affect the overall result. For example, $3 \times 4 = 4 \times 3$.

Example 3 *Recognising like terms*

Simplify the following expressions.

(a) $5st + 2ts$ (b) $-6q^2rp + 4prq^2$

Solution

(a) $5st + 2ts = 5st + 2st = 7st$

(b) $-6q^2rp + 4prq^2 = -6pq^2r + 4pq^2r = -2pq^2r$

The lower- and upper-case versions of the same letter are *different* symbols in mathematics. So, for example, $4y$ and $9Y$ are not like terms.

Any two constant terms are like terms. They can be collected using the normal rules of arithmetic.

Activity 13 *Recognising like terms*

Which of the following are pairs of like terms?

(a) $2ab$ and $5ab$ (b) $-2rst$ and $20rst$ (c) $3b$ and $6b^2$

(d) $5D$ and $5D$ (e) $2xy$ and $-3yx$ (f) $4c^2a$ and $9ac^2$

(g) z and $-z$ (h) abc and cba (i) $8c^2d$ and $9d^2c$

(j) $2A^2$ and $10a^2$ (k) $3fh$ and $3gh$ (l) 22 and -81

(m) 3 and $2m$

Often an expression contains several terms, some like and some unlike. You can simplify the expression by first changing the order of its terms so that the like terms are grouped together, and then collecting the like terms. This leaves an expression in which all the terms are unlike, which can't be simplified any further. Here's an example.

Example 4 *Collecting more like terms*

Simplify the following expressions.

(a) $2a + 5b - 7a + 3b$ (b) $12 - 4pq - 2q + 1 - qp - 2$

Solution

(a) Group the like terms, then collect them.

$$\begin{aligned} 2a + 5b - 7a + 3b &= 2a - 7a + 5b + 3b \\ &= -5a + 8b \end{aligned}$$

(b) Write qp as pq, group the like terms, then collect them.

$$\begin{aligned} 12 - 4pq - 2q + 1 - qp - 2 &= 12 - 4pq - 2q + 1 - pq - 2 \\ &= 12 + 1 - 2 - 4pq - pq - 2q \\ &= 11 - 5pq - 2q \end{aligned}$$

The unlike terms can be written in any order. For example, an alternative answer is $11 - 2q - 5pq$.

Activity 14 *Collecting more like terms*

You may find it helpful to mark the terms before rearranging them.

Simplify the following expressions.

(a) $4A - 3B + 3C + 5A + 2B - A$ (b) $-8v + 7 - 5w - 2v - 8$

(c) $20y^2 + 10xy - 10y^2 - 5y - 5xy$ (d) $-4ef + 8e^2f + 10fe - 3f^2e$

(e) $\frac{1}{2}a + \frac{1}{3}b + 2a + \frac{1}{4}b$

As you become more used to working with expressions, you'll probably find that you can collect like terms without grouping them together first. The worked examples in the course will usually do this, and you should do so too, as soon as you feel comfortable with it.

Sometimes when you collect two or more like terms, you find that the result is zero – that is, the terms *cancel each other out*. Here's an example.

Example 5 *Terms that cancel out*

Simplify the expression

$$M + 2N + 3M - 2N.$$

Solution

$$M + 2N + 3M - 2N = 4M + 0 = 4M$$

In the example above, $2N$ is added and then subtracted, and the addition and subtraction cancel each other out.

Activity 15 *Terms that cancel out*

Simplify the following expressions.

(a) $2a^3 - 3a - 2a^3 - 3a$ (b) $2m + n - 5m + 2n + 3m$

(c) $b + 2b + 3b - 6b$

Earlier in the unit, the formula

$$P = 1.24n - 0.69n$$

was found for a baker's profit, where P is the profit in £, and n is the number of loaves. Then the simpler formula

$$P = 0.55n$$

was found for the profit, by thinking about the situation in a different way. Notice that the simpler formula could have been obtained directly from the first formula, by collecting the like terms on the right-hand side:

$$P = 1.24n - 0.69n = 0.55n.$$

In the next activity there is another formula that can be simplified in this way.

Activity 16 *Collecting like terms in a formula*

A primary school parents' group is organising an outing to a children's activity centre, for c children and a adults, travelling by train.

(a) The cost of a ticket for the activity centre is £10 for a child and £2 for an adult. Find a formula for A, where £A is the total cost of admission to the activity centre for the group.

(b) The cost of a return train ticket to get to the activity centre is £7 for a child and £14 for an adult. Find a formula for T, where £T is the total cost of travel for the group.

(c) By adding your answers to parts (a) and (b), find a formula for C, where £C is the total cost of the trip for the group.

(d) Simplify the formula found in part (c) by collecting like terms, if you haven't already done so in part (c).

(e) Use the formula found in part (d) to find the cost of the trip for 22 children and 10 adults.