Information

The following questions will allow you to test how prepared you are for embarking on studying SM381. They assess the knowledge that we expect you to know already and be comfortable using.

Many of these questions have multiple algebraic or numerical variants, so you may get a different set of questions each time you attempt the quiz, which you may do as often as you wish.

When entering algebraic answers, you may use the symbols "+" (plus), "-" (minus), "*" (multiply), "/" (divide) and "^" (raise to the power of) as well as brackets "(" and ")". Use "i" for $\sqrt{-1}$, use "e" for the exponential function, and use "sqrt" to indicate the square root function. For Greek letters, simply type the name, such as "pi" for π or "theta" for θ.

When entering numerical answers you should also include units, if necessary, for example "3.2*m/s" or "6.8*kg*m^2/s^2".

Question 1 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

A component of a vector in Cartesian coordinates

$\mathbf{r} = 4\,\mathbf{e}_y + 2\,\mathbf{e}_z$ is a vector.

Give the value of the $\mathbf{e}_z$-component of $\mathbf{r}$.

$\mathbf{e}_z$-component =

The $\mathbf{e}_z$-component is the coefficient of $\mathbf{e}_z$ in the vector, $\mathbf{r}$. For $\mathbf{r} = 4\,\mathbf{e}_y + 2\,\mathbf{e}_z$ is it 2.

See An introduction to vector algebra Section 3.

Question 2 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

The magnitude of a vector in Cartesian coordinates

$\mathbf{r} = 2\,\mathbf{e}_x + \mathbf{e}_z$ is a vector.

Calculate the magnitude of vector $\mathbf{r}$.

Magnitude = .

(Enter sqrt(c) for the square root of c).

The magnitude of a vector written in the form

$\mathbf{r} = a_x\mathbf{e}_x + a_y\mathbf{e}_y + a_z\mathbf{e}_z$

is given by $a = \sqrt{a_x^2 + a_y^2 + a_z^2}$.

In this case, $a_x = 2, a_y = 0$ and $a_z = 1$ giving

$a = \sqrt{4+0+1} = \sqrt{5}.$

See An Introduction to Vector Algebra Section 3.

Question 3 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Unit vectors in Cartesian coordinates

$\mathbf{r} = 4\,\mathbf{e}_y + 2\,\mathbf{e}_z$ is a vector.

Calculate the unit vector $\hat{\mathbf{r}}$.

(Type ei to input $\mathbf{e}_x$, ey to input $\mathbf{e}_y$, ez to input $\mathbf{e}_z$ and sqrt(c) to input $\sqrt{c}$.)

$\hat{\mathbf{r}}$ = .

$\hat{\mathbf{r}}$ is the unit vector of $\mathbf{r}$ and is given by $\frac{1}{|\mathbf{r}|}\mathbf{r}$. It has the same direction as $\mathbf{r}$ and and magnitude of 1.

The magnitude of $\mathbf{r}$ is $|\mathbf{r}| = \sqrt{r_x^2 + r_y^2 + r_z^2}$.

In this case, $r_x = 0, r_y = 4$ and $r_z = 2$ giving $|\mathbf{r}| = \sqrt{0^2 + 4^2 + 2^2} = 2\sqrt{5}.$

This gives the unit vector $\frac{1}{|\mathbf{r}|}\mathbf{r} = \frac{1}{2\sqrt{5}}(4\,\mathbf{e}_y + 2\,\mathbf{e}_z)$

and so $\hat{\mathbf{r}} = \frac{2\,\mathbf{e}_y}{\sqrt{5}} + \frac{\mathbf{e}_z}{\sqrt{5}}$.

See An Introduction to Vector Algebra Section 2.

Question 4 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Scaling a vector

$\mathbf{a} = 2\,\mathbf{e}_x + \mathbf{e}_z$, write down $2\mathbf{a}$.

$2\mathbf{a}$ =

(Type ei for $\mathbf{e}_x$, ey for $\mathbf{e}_y$ and ez for $\mathbf{e}_z$.)

For a vector $\mathbf{a}$ and a (non-zero) scalar λ, the scalar multiple $\lambda\mathbf{a}$ is the vector whose magnitude is $|\lambda||\mathbf{a}|$, and has the same direction as $\mathbf{a}$ if $\lambda > 0$ or the opposite direction to $\mathbf{a}$ if $\lambda < 0$.

In this case, the scale factor is 2. Each component is multiplied by the scale factor, giving $2\mathbf{a} = 4\,\mathbf{e}_x + 2\,\mathbf{e}_z$.

See An Introduction to Vector Algebra Section 2.

Question 5 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Vector products

Determine the vector product $\mathbf{b}\times\mathbf{a}$ given $\mathbf{a} = \mathbf{e}_x - 5\,\mathbf{e}_y$ and $\mathbf{b} = 2\,\mathbf{e}_x - 3\,\mathbf{e}_z$.

(Type ei for $\mathbf{e}_x$, ey for $\mathbf{e}_y$ and ez for $\mathbf{e}_z$.)

$\mathbf{b}\times\mathbf{a}$ =

Using a determinant, the vector product is

$$\mathbf{b}\times\mathbf{a} = \begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ b_x & b_y & b_z \\ a_x & a_y & a_z \end{vmatrix} = \begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ 2 & 0 & -3 \\ 1 & -5 & 0 \end{vmatrix}$$

$= -15\,\mathbf{e}_x - 3\,\mathbf{e}_y - 10\,\mathbf{e}_z$.

See An Introduction to Vector Algebra 4.2.

Question 6 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Trigonometry

Given $f(x) = -\cos^2(x) + \sin^2(x)$, write $f(x)$ in terms of $\cos(2x)$.

The following equations are well-known trigonometrical identities:

$\cos^2(x) + \sin^2(x) = 1$,

$\cos^2(x) - \sin^2(x) = \cos(2x)$

and $\sin(2x) = \cos(x)\sin(x)$.

These can be combined and rearranged to give

$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$,

$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$

and $\sin^2(2x) = 1 - \cos^2(2x)$.

These equations can be used to show that

$$\begin{aligned} f(x) &= -\cos^2(x) + \sin^2(x) \\ &= \frac{-1 - \cos(2\,x)}{2} + \frac{1 - \cos(2\,x)}{2}. \\ &= -\cos(2\,x) \end{aligned}$$

See Algebra and other useful mathematical notation Section 4.

Question 7 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Finding the square modulus of a complex number

Given $z = 3 + 7\,\mathrm{i}$ where $\mathrm{i} = \sqrt{-1}$.

Determine $|z|^2$, the square modulus of z.

$|3 + 7\,\mathrm{i}|^2$ =

$|z|^2 = z^* z = (x - yi)(x + yi) = x^2 + xyi - xyi + y^2 = x^2 + y^2$.

Therefore, $|3 + 7\,\mathrm{i}|^2 = (3 - 7\,\mathrm{i})(3 + 7\,\mathrm{i}) = 9 + 49 = 58$.

See Complex numbers Section 1.

Question 8 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Finding the square of a complex number

Given $z = -7 + 4\,\mathrm{i}$ where $\mathrm{i} = \sqrt{-1}$.

Determine z^2, the square of z.

$(-7 + 4\,\mathrm{i})^2$ =

$z^2 = (x + yi)^2 = x^2 + 2xyi - y^2$.

Therefore, $z^2 = (-7 + 4\,\mathrm{i})^2 = 49 - 56\,\mathrm{i} - 16 = 33 - 56\,\mathrm{i}$

See Complex numbers Section 1.

Question 9 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

The polar form of complex numbers

Given the complex number $z = \frac{\sqrt{3}}{2} - \frac{i}{2}$ which has the Cartesian coordinates $(x, y) = (\frac{\sqrt{3}}{2}, -\frac{1}{2})$.

Determine the polar coordinates (r, θ) of z.

(Type pi for π.)

r =

θ =

The polar form of a complex number is $z = r(\cos\theta + \mathrm{i}\sin\theta)$.

where the radial coordinate, $r = \sqrt{x^2 + y^2}$ and the angular coordinate is θ. A unique value of θ is chosen with $-\pi < \theta \leq \pi$. θ can be calculated by solving $\tan\theta = y/x$ and making sure that θ is in the correct quadrant.

In this case, $r = \sqrt{(\frac{\sqrt{3}}{2})^2 + (-\frac{1}{2})^2} = 1$ and $\tan\theta = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$. Choosing θ in the correct quadrant gives $\theta = -\frac{\pi}{6}$.

See Complex numbers Section 2.

Question 10 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Complex numbers and Euler's formula

Given $z = e^{8\,\mathrm{i}\,x}$ where x is real.

Write down Re(z), the real part of z.

Re(z) =

Using Euler's formula, $z = Ae^{ikx} = A(\cos(kx) + \mathrm{i}\sin(kx))$ gives

$z = e^{8\,\mathrm{i}\,x} = \cos(8\,x) + \mathrm{i}\,\sin(8\,x)$,

so the real part of z is $\cos(8\,x)$.

See Complex numbers Section 3.

Question 11 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Complex numbers and Euler's formula

Given $z = 8\,e^{-4\,\mathrm{i}\,x}$ where x is real.

Write down Im(z), the imaginary part of z,

Im(z) =

Using Euler's formula, $z = Ae^{ikx} = A(\cos(kx) + \mathrm{i}\sin(kx))$ gives

$z = 8\,e^{-4\,\mathrm{i}\,x} = 8\,\cos(4\,x) - 8\,\mathrm{i}\,\sin(4\,x)$,

so the imaginary part of z is $-8\,\sin(4\,x)$.

See Complex numbers Section 3.

Question 12 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Cylindrical coordinates

Express the point given by Cartesian coordinates, $(x, y, z) = (1, 0, -1)$, in cylindrical coordinates (r, ϕ, z).

(Type pi for π.)

$r =$

$\phi =$

$z =$

The cylindrical coordinates (r, ϕ, z) of a point are related to the Cartesian coordinates (x, y, z) of the same point by $r = (x^2 + y^2)^{1/2}$, $\tan\phi = \frac{y}{x}$ and $\cos\phi = \frac{x}{r}$, and $z = z$.

In this case, $x = 1, y = 0$ and $z = -1$ giving $r = \sqrt{(1)^2 + (0)^2} = 1$, $\tan\phi = \frac{(0)}{(1)} = 0$ and $\cos\phi = \frac{(1)}{(1)} = 1$ giving $\phi = 0$, and $z = -1$.

See Coordinate systems Section 5.

Question 13 **Not answered**

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Spherical coordinates

Express the point given by Cartesian coordinates, $(x, y, z) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1)$, in spherical coordinates (r, θ, ϕ).

Your angles must be given in radians.

(Type sqrt(a) for $\sqrt{a}$ and pi for π.)

$r =$

$\theta =$

$\phi =$

The spherical coordinates (r, θ, ϕ) of a point are related to the Cartesian coordinates (x, y, z) of the same point by $r = (x^2 + y^2 + z^2)^{1/2}$, $\cos\theta = \frac{z}{r}$, $\tan\phi = \frac{y}{z}$ and $\cos\phi = \frac{x}{r\sin\theta}$.

In this case, $x = \frac{1}{\sqrt{2}}$, $y = \frac{1}{\sqrt{2}}$ and $z = 1$ giving

$r = \sqrt{(\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2 + (1)^2} = \sqrt{2}$, $\cos\theta = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$ giving $\theta = \frac{\pi}{4}$ and $\tan\phi = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

and $\cos\phi = \frac{\left(\frac{1}{\sqrt{2}}\right)}{\left(\sqrt{2}\sin(\frac{\pi}{4})\right)} = \frac{1}{\sqrt{2}}$ giving $\phi = \frac{\pi}{4}$.

See Coordinate systems Section 4.

Question 14 **Not answered**

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

The determinant of a matrix

Calculate the determinant of $\mathbf{A} = \begin{pmatrix} a & b & c \\ x & 0 & y \\ x & z & -y \end{pmatrix}$.

$\det \mathbf{A} =$

For a 3×3 matrix, the determinant is given by

$$\det \mathbf{A} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$
$$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1).$$

So for the matrix $\mathbf{A} = \begin{pmatrix} a & b & c \\ x & 0 & y \\ x & z & -y \end{pmatrix}$,

the determinant is $-a\,y\,z + c\,x\,z + 2\,b\,x\,y$.

See Matrices and determinants Section 4.

Question 15 **Not answered**

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Sum notation

Evaluate

(a)

$$S_1 = \sum_{j=1}^{4} j.$$

$S_1 =$

(b)

$$S_2 = \sum_{j=1}^{2} (j^2 + 1).$$

$S_2 =$

Given a set of numbers $a_1, a_2, a_3, \ldots, a_n$, then

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n.$$

Therefore,

$$\sum_{j=1}^{4} j = 1 + 2 + 3 + 4 = 10$$

and

$$\sum_{j=1}^{2} (j^2 + 1) = 2 + 5 = 7$$

See Algebra and other useful mathematical notation Section 5.

Question 16 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Differentiating using the product rule

Differentiate $y = \frac{\sin(x)}{a^2+x^2}$, with respect to x.

$\frac{dy}{dx} =$

If u and v are functions of x, the product rule gives

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}.$$

Using the product rule for $y = \frac{\sin(x)}{a^2+x^2}$, and letting $u = \frac{1}{a^2+x^2}$ and $v = \sin(x)$ gives

$$\frac{dy}{dx} = (\frac{1}{a^2+x^2})\frac{d}{dx}(\sin(x)) + (\sin(x))\frac{d}{dx}\left(\frac{1}{a^2+x^2}\right)$$

$$= \left(\frac{1}{a^2+x^2}\right)(\cos(x)) + \left(-\frac{2\,x}{(a^2+x^2)^2}\right)(\sin(x))$$

$$= \frac{\cos(x)}{a^2+x^2} - \frac{2\,x\,\sin(x)}{(a^2+x^2)^2}.$$

See Differentiation Section 2.3.

Question 17 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Differentiating an exponential function once

Differentiate $u(t) = 2\,e^{2\,i\,a\,t}$, with respect to t, where a is a constant and $i = \sqrt{-1}$.

$\frac{du}{dt} =$

Using the composite or chain rule with $u = 2\,e^{2\,i\,a\,t}$, we have

$$\frac{du}{dt} = 2\,e^{2\,i\,a\,t} \times \frac{d}{dt}(2\,i\,a\,t)$$

$$= 4\,i\,a\,e^{2\,i\,a\,t}.$$

See Differentiation Section 2.4.

Question 18 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Differentiating an exponential function twice

Given $u(t) = 3\,e^{-i\,a\,t}$, where a is a constant and $i = \sqrt{-1}$. Determine the second order differential, $\frac{d^2u}{dt^2}$.

$\frac{d^2u}{dt^2} =$

Using the composite or chain rule with $u = 3\,e^{-i\,a\,t}$, we have

$$\frac{du}{dt} = 3\,e^{-i\,a\,t} \times \frac{d}{dt}(-i\,a\,t)$$

$$= -3\,i\,a\,e^{-i\,a\,t}.$$

$$\frac{d^2u}{dt^2} = -3\,i\,a\frac{d}{dt}\left(e^{-i\,a\,t}\right) = -3\,i\,a\left(-i\,a e^{-i\,a\,t}\right)$$

$$= -3\,a^2\,e^{-i\,a\,t}.$$

The final simplication used $i^2 = -1$.

See Differentiation Section 2.4.

Question 19 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Linear differential equations

(a) Find the values of k for which $\cos(kt)$ and $\sin(kt)$ are solutions of the differential equation

$$\frac{d^2y}{dt^2} + 9y = 0.$$

$k =$ [ ] or [ ]

(b) Select a linear combination of the solutions in part (a) which is a general solution to the differential equation.

- (No answer given)
- $y = \alpha\,\cos(k\,t) + \beta\,\sin(k\,t)$
- $y = \beta\,\sin(k\,t)$
- $y = \alpha\,\cos(k\,t)$

(c) Select the property of the differential equation that guarantees that this linear combination is a solution.

(No answer given)

(a) Consider $y = \cos(kt)$, then

$$\frac{dy}{dt} = -k\sin(kt) \quad \text{and} \quad \frac{d^2y}{dt^2} = -k^2\cos(kt).$$

Substituting for y and $\frac{d^2y}{dt^2}$ into the original differential equation gives

$$-k^2\cos(kt) + 9\cos(kt) = 0.$$

This equation must be valid for all values of t, therefore,

$$-k^2 + 9 = 0$$
$$k^2 = 9$$
$$k = \pm 3.$$

Similarly for $y = \sin(kt)$, the solution is $k = \pm 3.$

(b) The solutions in part (a) are $y = \cos(3t)$ and $y = \pm\sin(3t)$. An arbitrary linear combination of them is $y = \alpha\,\cos(3\,t) + \beta\,\sin(3\,t)$ where α and β are arbitrary constants. As there are two arbitrary constants and this is a second order differential equation, this is a general solution.

(c) The fact that the differential equation is *linear* guarantees that this is a solution.

See Differential equations Sections 2 and 4.

Question 20 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Integration by substitution

Evaluate the integral

$$I = \int_0^q -\frac{r}{b\,r^2 + 4}\,dr,$$

where b and q are positive constants.

$I =$ [ ]

The method to use is *integration by substitution* and this is a definite integral.

Let $u = b\,r^2 + 4$, then $\frac{du}{dr} = 2\,b\,r$.

The limits of integration $r = 0$ and $r = q$ correspond to $u = 4$ and $u = b\,q^2 + 4$. Hence

$$\int_{r=0}^{r=q} -\frac{r}{b\,r^2+4}\,dr = \left(-\frac{1}{2\,b}\right)\int_{u=4}^{u=b\,q^2+4}\frac{1}{u}\,du$$
$$= \left(-\frac{1}{2\,b}\right)\left[\ln(u)\right]_{u=4}^{u=b\,q^2+4}$$
$$= \frac{\ln(4)}{2\,b} - \frac{\ln(b\,q^2+4)}{2\,b}.$$

See Integration 4.3 and 5.

Question 21 Not answered

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

The gradient of a scalar field

Consider $f = -z + 3\,z^2$.

Evaluate grad f.

(Type ei to input $\mathbf{e}_x$, ey to input $\mathbf{e}_y$ and ez to input $\mathbf{e}_z$.)

grad f = [ ]

The gradient of a scalar field, f is a vector field given by

$$\mathbf{F} = \operatorname{grad} f = \nabla f = \frac{\partial f}{\partial x}\,\mathbf{e}_x + \frac{\partial f}{\partial y}\,\mathbf{e}_y + \frac{\partial f}{\partial z}\,\mathbf{e}_z.$$

In this case, $f = -z + 3\,z^2$, so that,

$$\frac{\partial f}{\partial x} = 0, \ \frac{\partial f}{\partial y} = 0, \text{ and } \frac{\partial f}{\partial z} = -1 + 6\,z,$$

which gives the vector field

$$\mathbf{F} = (-1 + 6\,z)\,\mathbf{e}_z.$$

See Vector calculus and fields Section 7.

Question 22 **Not answered**

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

The divergence of a vector field in spherical coordinates

Consider the vector field

$$\mathbf{F} = \left(-\frac{\cos(\theta)}{r^2}\right)\,\mathbf{e}_r + (-r)\,\mathbf{e}_\theta + (-r\,\cos(\theta))\,\mathbf{e}_\phi$$

which is written in spherical coordinates, (r, θ, ϕ).

Evaluate the divergence of $\mathbf{F}$.

In your answer type r for coordinate r, theta for coordinate θ and phi for coordinate ϕ.

$\mathrm{div}\,\mathbf{F} =$ [ ]

In spherical coordinates

$$\mathrm{div}\,\mathbf{F} = \frac{1}{r^2}\frac{\partial(r^2\,F_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\,F_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi}$$

In this case $F_r = -\frac{\cos(\theta)}{r^2}$, $F_\theta = -r$ and $F_\phi = -r\,\cos(\theta)$,

giving

$$\frac{\partial(r^2\,F_r)}{\partial r} = \frac{\partial(r^2\,(-\frac{\cos(\theta)}{r^2}))}{\partial r} = \frac{\partial(-\cos(\theta))}{\partial r} = 0$$

and

$$\frac{\partial(\sin\theta\,F_\theta)}{\partial\theta} = \frac{\partial(\sin\theta\,(-r))}{\partial\theta} = \frac{\partial(-r\,\sin(\theta))}{\partial\theta} = -r\,\cos(\theta)$$

and

F_ϕ is independent of ϕ so that $\frac{\partial(F_\phi)}{\partial\phi} = 0.$

Bringing everything together gives

$$\begin{aligned}\mathrm{div}\,\mathbf{F} &= \frac{1}{r^2}(0) + \frac{1}{r\sin\theta}(-r\,\cos(\theta)) \\ &= -\frac{\cos(\theta)}{\sin(\theta)}.\end{aligned}$$

See Vector calculus and fields Section 3.

Question 23 **Not answered**

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

The curl of a vector field

Consider $\mathbf{A} = -2\,y\,z\,\mathbf{e}_x - 5\,x\,y\,\mathbf{e}_y + 4\,x^2\,\mathbf{e}_z$.

Find the curl of $\mathbf{A}$.

(Type ei to input $\mathbf{e}_x$, ey to input $\mathbf{e}_y$ and ez to input $\mathbf{e}_z$.)

curl $\mathbf{A}$= [ ]

$$\mathrm{curl}\,\mathbf{A} = \begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -2\,y\,z & -5\,x\,y & 4\,x^2 \end{vmatrix}$$

$$\begin{aligned} &= \left(\frac{\partial}{\partial y}(4\,x^2) - \frac{\partial}{\partial z}(-5\,x\,y)\right)\mathbf{e}_x + \left(\frac{\partial}{\partial z}(-2\,y\,z) - \frac{\partial}{\partial x}(4\,x^2)\right)\mathbf{e}_y + \left(\frac{\partial}{\partial x} \ldots\right. \\ &= ((0) - (0))\mathbf{e}_x + ((-2\,y) - (8\,x))\mathbf{e}_y + ((-5\,y) - (-2\,z))\mathbf{e}_z \\ &= (-8\,x - 2\,y)\,\mathbf{e}_y + (-5\,y + 2\,z)\,\mathbf{e}_z \end{aligned}$$

See Vector calculus and fields Section 5.

Question 24 **Not answered**

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Line integrals

Evaluate the line integral

$$I = \int_C \mathbf{E}\cdot\mathbf{dl},$$

where $\mathbf{E} = x^3\,\mathbf{e}_x + 2\,y\,\mathbf{e}_y + z^3\,\mathbf{e}_z$ and C is the path from $(0,0,0)$ to $(1,0,0)$ to $(1,1,0)$.

$I =$ [ ]

For the path from $(0,0,0)$ to $(1,0,0)$, a small directed element along the path is $\delta\mathbf{l} = \mathbf{e}_x\delta x.$

Then $\mathbf{E}\cdot\mathbf{dl} = x^3\,\mathrm{d}x$ and $I_1 = \int_0^1 x^3\,\mathrm{d}x = \frac{1}{4}.$

For the path from $(1,0,0)$ to $(1,1,0)$, a small directed element along the path is $\delta\mathbf{l} = \mathbf{e}_y\delta y.$

Then $\mathbf{E}\cdot\mathbf{dl} = 2\,y\,\mathrm{d}y$ and $I_2 = \int_0^1 2\,y\,\mathrm{d}y = 1.$

The total line integral is $I = I_1 + I_2 = \frac{1}{4} + 1 = \frac{5}{4}.$

See Vector calculus and fields Section 4.

Question 25 **Not answered**

Marked out of 1.00

Tidy STACK question tool | Question tests & deployed variants

Volume integrals

Evaluate the volume integral

$$I = \int_B 4r^2 \ \mathrm{d}V$$

using spherical coordinates, where B is a spherical volume of radius a, centred on the origin.

(Type pi to enter π).

$I =$

The integrand does not depend on the spherical coordinates θ and ϕ. Consequently, the quickest way of doing the integral is to split the sphere into many spherical shells. A typical spherical shell has radius r, thickness δr and volume $4\,\pi\,r^2\,\delta r$. The volume integral is then

$$\int_B 4r^2 \ \mathrm{d}V = \int_0^a 4r^2 \times 4\,\pi\,r^2 \ \mathrm{dr}$$

$$= 16\,\pi \int_0^a r^4 \ \mathrm{dr}$$

$$= \frac{16\,\pi\,a^5}{5}.$$

Alternatively, a method which would work in more general cases is:

$$\int_B 4r^2 \ \mathrm{d}V = \int_{r=0}^{r=a} \int_{\theta=0}^{\theta=\pi} \int_{\phi=0}^{\phi=2\pi} 4r^2 \times r^2 \sin\theta \, \mathrm{d}\phi \, \mathrm{d}\theta \, \mathrm{d}r$$

$$= \int_{r=0}^{r=a} \int_{\theta=0}^{\theta=\pi} 2\pi \times 4\,r^4 \sin\theta \, \mathrm{d}\theta \, \mathrm{d}r$$

$$= \int_0^a 2 \times 2\pi \times 4\,r^4 \ \mathrm{dr}$$

$$= 16\,\pi \int_0^a r^4 \ \mathrm{dr}$$

$$= \frac{16\,\pi\,a^5}{5}.$$

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