

S385 Diagnostic quiz

Am I ready to start on S385?

The diagnostic quiz below is designed to help you to answer the question “Am I ready to start S385”. If you are not ready for the module you may find the course materials too demanding, or even struggle to complete the module. Working through the quiz is a useful exercise for *all* prospective S385 students, including those who have already completed the recommended prior study (see Section 1 below). The quiz will serve as a reminder of some of the knowledge and skills that it is assumed you will bring either from OU Stage 2 science and mathematics modules or from other prior study. It includes a set of questions that test your existing knowledge of manipulating numbers and symbols, astronomy and physics, calculus and coding.

You may approach this as an “open-book” exercise – i.e. you are free to consult previous module material, reliable web resources or textbooks to recall previously taught material – but you should ensure that you have a secure understanding of the concepts and skills being tested. Solutions are provided to all of the questions to help you understand the expectations of the module.

If you find that you can answer nearly all of the questions, then it is likely that you are well prepared to take on S385. However, if you find that you have substantial difficulties with more than two questions in any section, then we strongly recommend that:

- (a) you consider taking one or more Stage 2 OU modules that will prepare you for S385 (see Section 1). If you are not able to do this, or if you have already taken these modules but need reminding of their contents, then
- (b) you should study the preparatory material for Stage 3 astrophysics and cosmology, found in the [Prepare and make a head start](#) section of Physics, Astronomy and Planetary Sciences subject site. We emphasise, however, that this is not a substitute for taking the Stage 2 modules themselves.

If, after working through this quiz, you are still unsure about whether S385 is the right module for you, we advise you to seek further help and advice from the Student Support Team.

1 Suggested prior study

S385 *Cosmology and the distant Universe* is a Stage 3 module in astrophysics that makes intellectual demands appropriate to the third year of a conventional degree. Astrophysics is a subject that relies on astronomy, physics, computing, and most importantly mathematics. To succeed with this module it is essential to have studied the appropriate level of mathematics, which includes differential and integral calculus. Modern astrophysics relies on computational methods, and so you are also expected to have some prior experience of using coding to manipulate scientific information, ideally using the Python language.

Before attempting S385 you are recommended to have good passes (pass 1 or 2) in Stage 2 modules in astronomy, physics and mathematics. The recommended Stage 2 modules for:

- R51 (BSc Physics),
- M06 (Master of Physics), or
- Q64 (Natural Sciences) Physics or Astronomy & Planetary Science pathway

should provide sufficient preparation. If you do not have a good pass in one or more of the Level 2 modules you are strongly recommended to reflect on areas of difficulty, and use this quiz to identify areas to work on. You will be also able to start S385 if you have taken fairly recently, and passed well, courses equivalent to HND standard in physics or mathematics, or studied to at least the second year of a degree in one of these subjects.

If you are coming to S385 without having studied any of the Level 2 science or mathematics modules recommended above, it is essential that you establish whether or not your background and experience give you a sound basis on which to tackle the module. This diagnostic quiz is designed to help you determine this.

2 Manipulating numbers and symbols

Mathematics is a vital tool in astrophysics – it provides the language in which ideas are expressed and processes are described. To study S385 you must be fluent in your ability to manipulate and solve algebraic equations, including the use of powers, roots, and reciprocals. You must be able to work with logarithmic (\log_{10} and \log_e) and trigonometric (\sin , \cos , \tan) functions and understand what vectors represent and how they are combined. You should also be comfortable with graphical representations of equations, and be able to interpret what they show, and be able to calculate and combine measurement uncertainties.

If you have difficulty with more than two of these questions, you are likely to find the level of mathematics in S385 extremely challenging, and are strongly recommended to study an introductory mathematics module such as MST124.

Question 1

Simplify the following expression to the greatest possible extent.

$$\frac{(a^3)^{1/6} \times a^{-2}}{a^{-1/3}/a^{1/2}}.$$

Question 2

Combine the following two equations to eliminate m_2 and obtain an expression for v_2 in terms of q , i , P , m_1 , G and π .

$$q = m_1/m_2$$

$$\frac{m_1^3 \sin^3 i}{(m_1 + m_2)^2} = \frac{Pv_2^3}{2\pi G}$$

Question 3

Assume $h = 6.626 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$, $G = 6.67 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$ and $c = 2.9979 \times 10^8 \text{ m s}^{-1}$.

(a) Determine the SI unit of the quantity defined by

$$A = \sqrt{\frac{hc}{2\pi G}}$$

(b) Calculate the numerical value of A and express your answer in scientific notation.

Question 4

Ten measurements are made of the wavelength of a spectral line in the spectrum of a star. The mean value of these measurements is 585 nm and their standard deviation is 6 nm.

- What uncertainty should be quoted for the mean wavelength?
 - Are the measurements consistent with a suspected true value for the wavelength of 591 nm?
 - If the mean value needed to be known with a precision of ± 1 nm, how many measurements of the wavelength would have to be made?
-

Question 5

A function $y(t)$ is believed to have the form $y(t) = at^n$, where a and n are unknown constants. Given a set of pairs of data y and t , what form of graph would you plot to enable you to determine the unknown constants?

Question 6

The shortest side of a right-angled triangle has a length of 5.0 cm and the smallest internal angle is equal to 0.395 radians.

- (a) What are the sizes of all the internal angles in degrees?
 - (b) What are the lengths of the other two sides of the triangle?
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3 Calculus

In addition to the basic mathematics in Section 2, you should also be comfortable with calculus representation (e.g. dx/dt and $\int y dx$) and be able to manipulate and solve differentials and integrals involving simple algebraic functions, exponentials and trigonometric functions. You should be familiar with methods such as the chain rule, product rule, integration by substitution and integration by parts.

If you have difficulty with more than two of these exercises, you should consider taking a mathematics module such as MST224.

Question 7

A graph is plotted of the speed $v(t)$ of a particle as a function of time t .

- (a) What physical quantity is signified by the gradient of the graph at a particular value of t and how may this be written as a differential function?
 - (b) What physical quantity is signified by the area under the graph between $t = t_1$ and $t = t_2$ and how may this be written as an integral function?
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Question 8

If $y(t) = 6 \sin(3t^2)$, use the chain rule to find dy/dt .

Question 9

If $y(x) = 2x^3/(x + 3)^4$, use the product rule to find dy/dx .

Question 10

The pressure, volume and temperature in an interstellar gas cloud are related by $PV = NkT$, where N and k are constants, but P , V and T may evolve with time as the cloud expands or contracts. Use the technique of logarithmic differentiation with respect to time to find an expression for \dot{T}/T , where \dot{T} is a shorthand for dT/dt .

Question 11

Approximate the function $f(x) = \exp(3x)$ using a second-order Maclaurin series expansion.

Question 12

If ρ is a scalar quantity describing the density inside a gas cloud, write down an expression for $\nabla\rho$ and explain what it signifies.

Question 13

Evaluate the indefinite integral $\int \left(\frac{2}{x} + 3x^3\right) dx$

Question 14

Evaluate the definite integral $\int_0^1 (x^2 - 1)^4 2x dx$ by using the substitution $u = x^2 - 1$.

Question 15

If $\rho(r, \phi, \theta)$ is a function describing the density inside a star using spherical polar coordinates, explain what is signified by the multiple integral

$$\int_{r=0}^{r=R} \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi} \rho(r, \phi, \theta) r^2 \sin \theta dr d\phi d\theta$$

4 Astronomy and physics

Before embarking on S385, you should be familiar with some of the general terminology of astronomy, including the basic properties of stars and galaxies and methods to observe them. The physics knowledge required for S385 includes awareness of general concepts such as forces, energy, power and momentum, and familiarity with Newton's laws of motion and of gravity. You should recognize basic concepts in electricity and magnetism and properties of matter, particularly gases. You should be familiar with general concepts in quantum physics, such as photons, energy levels and wave-particle duality, and with the idea of emission line, absorption line, and continuous spectra and the information which they convey.

If you have difficulty with more than two of these questions, you should consider taking an introductory astronomy module such as S284 and/or a physics module such as S217.

Question 16

The star Regulus has a mass of 1.0×10^{31} kg, a radius of 2.45×10^9 m and a luminosity of 1.7×10^{29} W. Express the mass, radius and luminosity of this star in solar units. (You may assume that $M_{\odot} = 2.0 \times 10^{30}$ kg, $R_{\odot} = 7.0 \times 10^8$ m and $L_{\odot} = 3.8 \times 10^{26}$ W.)

Question 17

Outline the key events in the life cycle of a star and briefly compare the properties of Sun-like (main sequence) stars, red giants, white dwarfs and neutron stars.

Question 18

Briefly outline the structure of our galaxy, the Milky Way, and describe its overall contents. State how other galaxies are classified according to their morphology.

Question 19

The radio galaxy 3C 31 has a redshift of $z = 0.0169$. Explain how redshift is measured, and calculate the apparent speed of recession of the galaxy in km s^{-1} .

Question 20

Determine the mass of the Sun, given that the orbital radius of the planet Mercury is 5.79×10^{10} m and that its planetary year lasts 88.0 Earth days. ($G = 6.67 \times 10^{-11}$ N m² kg⁻².)

Question 21

Consider a fixed mass of an ideal gas.

- What happens to the pressure exerted by the gas if it is allowed to expand whilst maintaining a constant temperature?
 - What happens to the pressure exerted by the gas if its temperature is increased whilst maintaining a constant volume of the gas?
 - How do the average energy and average speed of the gas molecules alter if the absolute temperature is doubled?
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Question 22

- A hydrogen atom makes a transition from the $n = 5$ energy level to the $n = 1$ energy level. What is the energy of the photon that is emitted?
 - A hydrogen atom absorbs a photon of energy 1.89 eV. Between which two energy levels does it make a transition?
 - What happens if a hydrogen atom in its ground state absorbs a photon whose energy is 15.0 eV?
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Question 23

The unstable isotope of carbon represented by $^{14}_6\text{C}$ undergoes β^- -decay. Write down a balanced equation to describe this process, indicating what nucleus is formed as a result.

Question 24

A beam of electromagnetic radiation has a frequency of 1.20×10^{20} Hz.

- (a) What is the wavelength of this radiation?
- (b) What is the energy, in electronvolts, of the photons of which the beam is composed?
- (c) For what temperature of black body spectrum would the mean photon energy have this value?
- (d) Which part of the electromagnetic spectrum corresponds to radiation of this photon energy?

(You may assume $c = 3.00 \times 10^8$ m s⁻¹, $h = 6.63 \times 10^{-34}$ J s, $k = 1.38 \times 10^{-23}$ J K⁻¹ and $1 \text{ eV} = 1.60 \times 10^{-19}$ J.)

5 Coding

S385 involves the use of Python coding to explore physical models and analyse scientific data. Although we provide extensive teaching and resources to support this element of the module, you should have some familiarity with programming, ideally in Python, in advance of studying S385. We do not expect you to have memorised particular Python methods or syntax, so as with all of the questions in the quiz you should feel free to use internet or other resources to help in answering the questions.

If you have difficulty with more than two of these questions, you should consider taking a module with a coding element, such as SXPS288, or you should work through the “Programming in Python” material in the “Software and tools for Physical Sciences” area of the S-PHYSICS study site in advance of the module start.

Question 25

Write a short piece of Python code to print out the value of the function $\tan(\theta)$ for values of the angle θ between 0 and 180 degrees at 30 degree intervals.

Question 26

Below is a code snippet that calculates the luminosity of a set of astronomical objects based on an input file (in comma-separated variable format) containing distance and flux measurements. Write an explanation of what each line of the code does.

```
import numpy as np
import pandas as pd

def convert_kpc(darr):
    d = darr*(3.09e19) # 1 kpc = 3.09e19 m
    return d

def calc_lum(flux, dist):
    l = flux*4.0*np.pi*(dist**2)
```

```

    return l

df=pd.read_csv("flux_dist.csv")

fluxes=df["Flux"]
dists=df["Dist"]

mdists = convert_kpc(dists)

lumin = calc_lum(fluxes, mdists)

print("List of luminosities:")
for elem in lumin:
    print(f'{elem:.3e} W')

```

Question 27

Modify the code from the previous Question to make a plot of distance vs luminosity for the same (imaginary) dataset. Your plot should have logarithmic axes, because astronomical luminosities and distances typically span large ranges. The plot should also have suitable axis labels.

6 References

In addition to the resources provided on the S385 preparation website you may find some of the following books useful.

Basic Mathematics for the Physical Sciences, by R. Lambourne and M. Tinker, John Wiley and Sons, 2000 (ISBN 0 4718 5207 4).

Further Mathematics for the Physical Sciences, by M. Tinker and R. Lambourne, John Wiley and Sons, 2000 (ISBN 0 4718 6723 3).

The fundamentals of physics extended, (6th ed.), by D. Halliday, R. Resnick and J. Walker, John Wiley and Sons, 2013, (ISBN 10 1118230728)

Physics, by J. Cutnell and K. Johnson, John Wiley and Sons, 2014, (ISBN 10 111865188X)

University Physics, by H. Benson, John Wiley and Sons, 1996, (ISBN 0 4710 0689 0)

Universe, (8th ed.), by Roger Freedman and William J. Kaufmann, W. H. Freeman, 2007, (ISBN 978 0 7167 8584 2)

An introduction to modern astrophysics, by B. W. Carroll and D. A. Ostlie, Addison Wesley Longman Publishing Co., 2013, (ISBN 10 1292022930)

The dynamic Universe, (4th ed.), by T. P. Snow, West Publishing Company, 1991, (ISBN 0 3147 7104 2)

An introduction to the Sun and stars, by Simon F. Green and Mark H. Jones, Cambridge University Press, 2015, (ISBN 9781 107 49263 9)

An introduction to galaxies and cosmology, by Mark H. Jones, Robert J. A. Lambourne and Stephen Serjeant, Cambridge University Press, 2015, (ISBN 978 1 107 49261 5)

An introduction to the Solar System, by Neil McBride and Iain Gilmour,
Cambridge University Press, 2011, (ISBN 978 1 107 600928)

An introduction to astrobiology, by Iain Gilmour and Mark A. Sephton,
Cambridge University Press, 2011, (ISBN 978 1 107 600935)

Solutions to questions

Solution to Question 1

$$\begin{aligned}\frac{(a^3)^{1/6} \times a^{-2}}{a^{-1/3}/a^{1/2}} &= \frac{a^{3/6} \times a^{1/3}}{a^2 \times a^{-1/2}} = \frac{a^{1/2} \times a^{1/3} \times a^{1/2}}{a^2} \\ &= a^{(1/2)+(1/3)+(1/2)-2} = a^{(3+2+3-12)/6} = a^{-4/6} = a^{-2/3}\end{aligned}$$

Solution to Question 2

From the first equation, $m_2 = m_1/q$, and substituting this into the second equation:

$$\frac{m_1^3 \sin^3 i}{\left(m_1 + \frac{m_1}{q}\right)^2} = \frac{Pv_2^3}{2\pi G}$$

Dividing both the top and bottom lines of the left-hand side of this by m_1^2 gives

$$\frac{m_1 \sin^3 i}{[1 + (1/q)]^2} = \frac{Pv_2^3}{2\pi G}$$

which may be rearranged to end up with

$$v_2 = \left(\frac{2\pi G m_1}{P[1 + (1/q)]^2} \right)^{1/3} \sin i$$

Solution to Question 3

- (a) Both the digit 2 and the constant π are dimensionless, i.e. they have no SI unit. So putting the units for h , c and G into the equation, the SI unit of A must be

$$\sqrt{\frac{\text{kg m}^2 \text{s}^{-1} \times \text{m s}^{-1}}{\text{kg}^{-1} \text{m}^3 \text{s}^{-2}}} = \sqrt{\frac{\text{kg m}^3 \text{s}^{-2}}{\text{kg}^{-1} \text{m}^3 \text{s}^{-2}}}$$

and then m^3 and s^{-2} cancel on the top and bottom lines, leaving

$$\sqrt{\frac{\text{kg}}{\text{kg}^{-1}}} = \sqrt{\text{kg}^2} = \text{kg}$$

and the SI unit of A is kg.

- (b) Putting in the numbers:

$$A = \sqrt{\frac{6.626 \times 10^{-34} \times 2.9979 \times 10^8}{2\pi \times 6.67 \times 10^{-11}}} \text{ kg} = 2.1771 \times 10^{-8} \text{ kg}$$

Since the values in the question are given to varying numbers of decimal places, the answer above has too high a precision. An appropriate final answer is therefore $A = 2.18 \times 10^{-8} \text{ kg}$.

Solution to Question 4

- (a) The uncertainty σ_m in the mean value of n measurements is related to the standard deviation s_n of the measurements by $\sigma_m = s_n/\sqrt{n}$. So $\sigma_m = 6 \text{ nm}/\sqrt{10} \sim 2 \text{ nm}$. Note that this is much smaller than the uncertainty in a single measurement, which is represented by the standard deviation of 6 nm.
- (b) The difference between the mean value (585 nm) and the suspected true value (591 nm) is 6 nm, which is three times larger than the uncertainty in the mean. Assuming that values of the means that would be obtained from many sets of ten measurements have a Gaussian distribution, then the probability of the value of the mean differing from the true value by three times the uncertainty in the mean is only 0.003. It is therefore unlikely, though possible, that the true value is 591 nm.
- (c) If $\sigma_m = 1 \text{ nm}$, and $s_n = 6 \text{ nm}$, then $\sqrt{n} = s_n/\sigma_m = 6$, and so $n = 36$. So reducing the uncertainty from 2 nm to 1 nm would require almost four times as many measurements.

Solution to Question 5

Taking logarithms (to the base 10) of each side of the proposed equation yields $\log y = \log a + n \log t$. So plotting a graph of $\log y$ against $\log t$ would yield a straight line whose intercept on the vertical axis is equal to $\log a$ and whose gradient is equal to n . Hence both a and n could be determined from the graph.

Solution to Question 6

- (a) An angle of 0.395 radians can be converted into degrees as follows. Since π radians is equal to 180° , the angle in question is $(0.395 \text{ radians}) \times (180^\circ)/(\pi \text{ radians}) = 22.6^\circ$. One of the other internal angles is a right angle (90°), so since the internal angles of a triangle add up to 180° , the third angle is $180^\circ - 90^\circ - 22.6^\circ = 67.4^\circ$.
- (b) Let the hypotenuse of the triangle be of length h and the other unknown side be of length b . The smallest angle will be opposite the smallest side of the triangle, so $\sin 22.6^\circ = 5.0 \text{ cm}/h$ and therefore $h = 13.0 \text{ cm}$. Similarly, $\tan 22.6^\circ = 5.0 \text{ cm}/b$, so $b = 12.0 \text{ cm}$.

Solution to Question 7

- (a) The gradient of a graph of speed against time at a particular value of t is the magnitude of the acceleration (or the negative of the magnitude if the gradient is negative, i.e. a deceleration) of the particle at that instant of time. In symbols, $a(t) = dv(t)/dt$.
- (b) The area under a graph of speed against time between two limits is the distance s covered by the particle between these two times. In symbols $s = \int_{t_1}^{t_2} v(t) dt$.

Solution to Question 8

Put $u = 3t^2$ then $du/dt = 6t$. Also, $y = 6 \sin u$ so $dy/du = 6 \cos u = 6 \cos(3t^2)$.

Now, using the chain rule, $dy/dt = dy/du \times du/dt$, we have $dy/dt = 6 \cos(3t^2) \times 6t = 36t \cos(3t^2)$.

Solution to Question 9

First, put $u = 2x^3$ and $v = (x + 3)^{-4}$, then $y = uv$ and we can use the product rule:

$$\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}.$$

It is straightforward to find $du/dx = 6x^2$, and $dv/dx = -4(x + 3)^{-5}$. Finally putting this all together,

$$\begin{aligned} \frac{dy}{dx} &= \left[2x^3 \times -4(x + 3)^{-5} \right] + \left[(x + 3)^{-4} \times 6x^2 \right] \\ &= \frac{-8x^3}{(x + 3)^5} + \frac{6x^2}{(x + 3)^4} \end{aligned}$$

Solution to Question 10

Taking natural logarithms of each side of the equation,

$$\log_e P + \log_e V = \log_e Nk + \log_e T$$

Now taking the time derivative of this expression, noting that in general $d(\log_e x)/dt = \dot{x}/x$, and that N and k are constants, we have

$$\frac{\dot{P}}{P} + \frac{\dot{V}}{V} = \frac{\dot{T}}{T}$$

Solution to Question 11

The first and second derivatives of the function are $f'(x) = 3 \exp(3x)$ and $f''(x) = 9 \exp(3x)$. So the second-order Maclaurin series expansion is

$$\exp(3x) = f(0) + xf'(0) + x^2 f''(0)/2!$$

$$\exp(3x) = \exp 0 + x(3 \exp 0) + x^2(9 \exp 0)/2$$

$$\exp(3x) = 1 + 3x + 9x^2/2$$

Solution to Question 12

In terms of components

$$\nabla \rho = \left(\frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y}, \frac{\partial \rho}{\partial z} \right).$$

$\nabla \rho$ is a vector field which describes the density gradient within the star.

Solution to Question 13

$$\int \left(\frac{2}{x} + 3x^3 \right) dx = 2 \log_e x + \frac{3x^4}{4} + C$$

Solution to Question 14

Setting $u = x^2 - 1$, clearly $du/dx = 2x$ or $du = 2x dx$. Substituting these into the original integral gives

$$\int_0^1 (x^2 - 1)^4 2x dx = \int_{x=0}^{x=1} u^4 du$$

Now, evaluating this integral is straightforward:

$$\int_{x=0}^{x=1} u^4 du = \left[\frac{u^5}{5} \right]_{x=0}^{x=1}$$

We now reverse the original substitution to give

$$\left[\frac{(x^2 - 1)^5}{5} \right]_{x=0}^{x=1} = 0 - (-1)^5/5 = 1/5$$

Solution to Question 15

This is a volume integral, which sums the product of the density and infinitesimal volume at each position. Since the product of density and volume is mass, evaluating this integral will give the total mass of the star.

Solution to Question 16

The mass of Regulus is $(1.0 \times 10^{31} \text{ kg}) / (2.0 \times 10^{30} \text{ kg } M_{\odot}^{-1}) = 5.0 M_{\odot}$.

The radius of Regulus is $(2.45 \times 10^9 \text{ m}) / (7.0 \times 10^8 \text{ m } R_{\odot}^{-1}) = 3.5 R_{\odot}$.

The luminosity of Regulus is $(1.7 \times 10^{29} \text{ W}) / (3.8 \times 10^{26} \text{ W } L_{\odot}^{-1}) = 450 L_{\odot}$.

Solution to Question 17

Stars are formed by the collapse of fragments of dense molecular clouds. When the central regions become hot enough for nuclear fusion to be initiated, the star is born and joins what is known as the “main sequence” in reference to the standard Hertzsprung-Russell diagram used to study stellar life cycles. The star remains on the main sequence whilst undergoing hydrogen fusion in its core by the proton–proton chain or the CNO cycle. When hydrogen in the core is exhausted, helium fusion may begin. Other nuclear fusion reactions are subsequently possible in massive stars. When nuclear fuel is exhausted the star ends its life in one of several ways, depending on its mass. Low mass stars shed their outer layers as planetary nebulae and the core collapses to form a white dwarf. Massive stars explode as supernovae and the core collapses to form a neutron star or black hole.

Red giants are stars much larger than the Sun, in the late stages of hydrogen burning or helium burning. White dwarfs and neutron stars are the end states for low and high mass stars, respectively, and are very compact objects in which the matter exists in a very dense state known as degeneracy in which quantum mechanical effects become important.

Solution to Question 18

The Milky Way is a typical spiral galaxy comprising a disc, a halo and a nuclear bulge. Young, high metallicity, population I stars are found mainly in the spiral arms, and somewhat older population I stars are found throughout the disc. Older, low metallicity, population II stars are found in the halo (including globular clusters) and the nuclear bulge. HII regions (ionized hydrogen) and open star clusters are found in spiral arms and are associated with star formation.

Other galaxies are classified according to their shape as elliptical, lenticular, spiral or irregular.

Solution to Question 19

Redshift is the shift in wavelength (or frequency) of radiation emitted by an astronomical object that is receding from us due to the Universe's expansion. The spectra of astronomical objects typically contain emission (and/or absorption) lines - redshift is measured by comparing the observed wavelength of these lines with their expected laboratory values. For small redshifts, the speed of recession is related to the redshift by $v = zc$. So in this case $v = 0.0169 \times 3.00 \times 10^5 \text{ km s}^{-1} = 5070 \text{ km s}^{-1}$.

Solution to Question 20

A planet of mass m moving in a circular orbit of radius r with uniform angular speed ω must be subject to a centripetal force of magnitude $F = mr\omega^2$. If this force is supplied by the gravitational attraction of the Sun, then by Newton's law of universal gravitation,

$$mr\omega^2 = \frac{GM_{\odot}m}{r^2}$$

For a planet with angular speed ω , the orbital period is $P = 2\pi/\omega$, so replacing ω in the equation above by $2\pi/P$ gives

$$\frac{(2\pi)^2 mr}{P^2} = \frac{GM_{\odot}m}{r^2}$$

Rearranging and cancelling the common terms, this yields

$$M_{\odot} = \frac{(2\pi)^2 r^3}{GP^2}$$

Putting in the numbers we have,

$$\begin{aligned} M_{\odot} &= \frac{(2\pi)^2 \times (5.79 \times 10^{10} \text{ m})^3}{(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}) \times (88.0 \times 24 \times 3600 \text{ s})^2} \\ &= 1.99 \times 10^{30} \text{ kg} \end{aligned}$$

Solution to Question 21

- (a) Using the ideal gas equation, $PV = NkT$, if V increases whilst N and T are kept fixed, then clearly P must decrease. So the pressure exerted by the gas will decrease.
- (b) Using the ideal gas equation again, if T increases whilst N and V are kept fixed, then clearly P must increase. So the pressure exerted by the gas will increase.
- (c) The average energy of the molecules is proportional to the absolute temperature and the average speed of the molecules is proportional to the square root of the absolute temperature. So if the temperature is doubled, the average energy also doubles, whereas the average speed increases by a factor of $\sqrt{2}$.

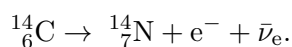
Solution to Question 22

The energy levels of the hydrogen atom are determined by the equation $E_n = -13.60 \text{ eV}/n^2$, where n is an integer. The energies of the first six energy levels are therefore: $E_1 = -13.60 \text{ eV}$, $E_2 = -3.40 \text{ eV}$, $E_3 = -1.51 \text{ eV}$, $E_4 = -0.85 \text{ eV}$, $E_5 = -0.54 \text{ eV}$ and $E_6 = -0.38 \text{ eV}$.

- (a) In making a transition from the $n = 5$ energy level to the $n = 1$ energy level, the atom emits a photon of energy $(-0.54 \text{ eV}) - (-13.60 \text{ eV}) = 13.06 \text{ eV}$.
- (b) In order to absorb a photon of energy 1.89 eV , a hydrogen atom must make a transition *from* the $n = 2$ energy level *to* the $n = 3$ energy level.
- (c) The atom is ionized. The first 13.6 eV is used to raise the atom from the ground-state energy level to a state in which the nucleus (proton) and electron are widely separated. The remaining 1.4 eV is transferred to the proton and electron as kinetic energy.

Solution to Question 23

β^- -decay occurs when a neutron transforms into a proton. So the mass number of the nucleus remains the same, but its atomic number increases by one. The balanced equation is



The resulting nucleus is an isotope of nitrogen and an electron (β -particle) and an electron antineutrino are emitted, carrying away the released energy.

Solution to Question 24

- (a) Using $c = \lambda\nu$, the wavelength of the radiation is

$$\begin{aligned}\lambda &= (3.00 \times 10^8 \text{ m s}^{-1}) / (1.20 \times 10^{20} \text{ Hz}) \\ &= 2.50 \times 10^{-12} \text{ m or } 0.00250 \text{ nm}.\end{aligned}$$

- (b) Using $E_{\text{ph}} = h\nu$, the energy of the photons of which this radiation is composed is

$$\begin{aligned}E_{\text{ph}} &= (6.63 \times 10^{-34} \text{ J s}) \times (1.20 \times 10^{20} \text{ Hz}) \\ &= 7.96 \times 10^{-14} \text{ J}.\end{aligned}$$

Converting this into electronvolts:

$$\begin{aligned} E_{\text{ph}} &= (7.96 \times 10^{-14} \text{ J}) / (1.60 \times 10^{-19} \text{ J eV}^{-1}) \\ &= 4.98 \times 10^5 \text{ eV or about 500 keV.} \end{aligned}$$

- (c) Using $\langle E_{\text{ph}} \rangle = 2.70kT$, a black body spectrum whose mean photon energy is 500 keV would have a temperature of

$$\begin{aligned} T &= \langle E_{\text{ph}} \rangle / 2.70k \\ &= (7.96 \times 10^{-14} \text{ J}) / (2.70 \times 1.38 \times 10^{-23} \text{ J K}^{-1}) \\ &= 2.14 \times 10^9 \text{ K} \end{aligned}$$

or just over 2 billion kelvin.

- (d) Photons with this energy lie close to the boundary between the X-ray and gamma-ray parts of the electromagnetic spectrum.

Solution to Question 25

A possible solution is shown below:

```
import numpy as np

# define list of angles
thetas=[0,30,60,90,120,150,180]

for ang in thetas:
    # convert to radians
    thetar = ang*np.pi/180.0
    tantheta=np.tan(thetar)
    print thetar*180.0/np.pi,tantheta
```

This solution outputs a list of pairs of angles and their tan values.

Solution to Question 26

The first line of the code imports the numpy library. The next three lines define a function, `convert_kpc`, that takes one argument (“darr”) and multiplies it by a conversion factor, returning the result. The subsequent three lines define a second function, `calc_lum`, that calculates luminosity based on two input values or arrays containing the flux and distance. The next line represents the start of the main body of the code. The input file is read into a `pandas` DataFrame, and then two variables, `fluxes` and `dists` are defined and allocated to arrays that each contain the relevant data column. The next two lines consist of operations using the two data arrays: first the distances array is converted to units of metres using the `convert_kpc` function, and then the output of this step together with the fluxes array are used as input for the `calc_lum` function, which returns the list of luminosities.

Finally the list of luminosities is printed out, with the specific syntax of the print statement ensuring that the output is in scientific format, to 3 decimal places.

Solution to Question 27

The example solution below uses the `matplotlib` library, which is the most widely used plotting package for Python – a solution that uses a different library (e.g. `bokeh`) would also be acceptable.

An additional import statement needs to be added near the start of the code, e.g.:

```
import matplotlib.pyplot as plt
```

and then some additional lines to produce the requested plot should be added at the end.

```
plt.figure()
plt.scatter(dists, lumin)
plt.xscale("log")
plt.yscale("log")
plt.xlabel("Distance (kpc)")
plt.ylabel("Luminosity (W)")
```

Alternative plotting methods (e.g. using the `axes` matplotlib functionality) would also be acceptable.