**SDK100 Science and health: an evidence-based approach**

# SDK100 Maths Workbook

## Contents

# 1. Introduction

This workbook collects together the general aspects of maths you will use during the module. **You are not required to study the material in this workbook**, it is intended as a reference resource that you can use alongside the module materials if you wish to refresh your memory, increase your confidence or find out more about some aspects of maths. You may also find some of this general maths advice useful later in your studies.

The workbook includes some extra practice questions – the answers to these are in the back of the workbook. You are free to use a calculator whenever you need to in SDK100, but in some cases the practice questions suggest working out the answers by hand to make sure you understand the method.

## 1.1 Answering maths questions

There are three things you should be careful to do when answering any maths questions in your OU studies.

**(a) Write down all of your workings**

Always write down all your workings as well as the final answer. If you are answering a maths question in a TMA your workings will help your tutor to see what you did right, and, if you make a mistake, where you went wrong. Secondly, most of the marks awarded for maths questions in assessments are likely to be for the intermediate steps of working (i.e. the method you have used) and not for the final answer. If your answer is written down in a methodical way you are also less likely to make a mistake and more likely to get the correct answer. It is perfectly acceptable to write words of explanation in your answer – it doesn't all have to be in symbols and numbers!

**(b) Always include units**

Throughout this module the importance of giving appropriate units with all quantities is emphasised. So, when you work out the value of something, each line of your calculation should include units as well as numbers.

Before you carry out a calculation, make sure that the values you have been given are in units that are consistent with each other. For example, it doesn't make sense to add one length given in centimetres (cm) to another length given in metres (m). You should first convert all values to the same unit before doing a calculation.

**(c) Ensure your answer makes sense**

Check whether your answer is about the size you expected with respect to the values you used in the calculation – this won't always guarantee that your answer is correct, but will sometimes highlight when mistakes have been made.

# 2. Doing calculations with and without a calculator

For this module it is assumed that you can add, subtract, multiply and divide whole numbers.

- Check that you can do the following four calculations without using a calculator:
  $9 + 3; 9 - 3; 9 \times 3; 9 \div 3$.

- You should have obtained the following answers:
  $9 + 3 = 12; 9 - 3 = 6; 9 \times 3 = 27; 9 \div 3 = 3$.

Now, check that you can use your calculator to obtain the same answers. You are likely to need to enter the numbers and symbols in a simple sequence from left to right (e.g. '9' then '+' then '3') and then to press the '=' key. However, some calculators use the symbol '*' instead of '×' for multiplication and '/' instead of '÷' for division, or may require the user to press 'Enter' instead of '=' to get the answer.

## 2.1 Doing calculations in the right order

Many scientific calculations are rather more complicated than those discussed above, in that they involve several steps. In some of these calculations, for example $3 + 4 - 2$ or $6 \times 2 \div 3$ you simply need to start at the left and work through to the right. So in the first example, adding 3 and 4 gives 7, then subtracting 2 gives the final answer of 5. But things are not always so straightforward.

Consider the calculation $3 + 2 \times 4$. If you simply work from left to right in this case, adding 3 and 2 gives 5, then multiplying by 4 gives 20, but this is the incorrect answer.

There is a rule, applied by mathematicians and scientists everywhere, which states: multiplication and division should be carried out before addition and subtraction.

Applying this rule to the calculation $3 + 2 \times 4$, the multiplication of 2 and 4 should be done first, giving 8, then the 3 is added to give the correct final answer of 11. Most modern calculators 'know' this rule (which is known as a rule of precedence, where precedence means 'priority'), so entering $3 + 2 \times 4$ into your calculator in exactly the order in which it is written should give the correct answer of 11. Check this now.

### Practice questions 1

Do the following calculations without using your calculator. Then use your calculator to check the answers.

(a) $3 \times 4 + 2$

(b) $2 + 4 \times 3$

(c) $4 \times 2 - 21 \div 7$

(d) $7 + 9 \div 2$

## 2.2 What are powers?

Most people are familiar with the fact that $5 \times 5$ can also be written as $5^2$ ('five squared') and $4 \times 4 \times 4$ as $4^3$ ('four cubed'). This shorthand notation can be extended indefinitely, so $2 \times 2 \times 2 \times 2 \times 2 \times 2$ becomes $2^6$ (said as 'two to the power of six', or more usually just 'two to the six'). In this example, the 2 is called the *base number* and the superscript 6 (indicating the number of twos that have been multiplied together) is called the *power* or the *exponent*.

If you have a scientific calculator you can use your calculator's instruction booklet to find out how to enter a base number raised to a power, such as $2^6$. You may have a button labelled '$x^y$', in which case you should press '2' then '$x^y$' then '6'. Alternatively, you may have a button labelled '^' in which case you should press '2' then '^' then '6'. (Note that $2^6 = 64$; your calculator may calculate this value straight away.)

If you are asked to calculate, say, $5 \times 3^2$ there is another rule of precedence.

Powers should be calculated before multiplication, division, addition or subtraction.

So, in the example of $5 \times 3^2$ the $3^2$ should be calculated first: $3^2 = 3 \times 3 = 9$ so $5 \times 3^2 = 5 \times 9 = 45$.

A negative sign in front of a power has a special meaning. It denotes 1 divided by the number raised to the power, so for example $3^{-2}$ means $1 \div 3^2 = \frac{1}{3^2} = \frac{1}{9} = 0.111$ (recurring). To enter a base number raised to a negative power into your calculator, you may need to use the button labelled something like $+/-$ or $\pm$ in order to enter the negative power.

So far, the four arithmetic operations have been written as $+$, $-$, $\times$ and $\div$. However, in scientific calculations, division is more usually written as a fraction (see Section 4 for more about fractions). Twelve divided by three could equally accurately be written as $12 \div 3$, $12/3$ or $\frac{12}{3}$.

To calculate $8 \times 2^{-2}$, the $2^{-2}$ should be calculated first: $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

then $8 \times \frac{1}{4} = \frac{8}{4} = 2$.

## 2.3 The role of brackets

Sometimes you need to do the addition or subtraction in a calculation before the multiplication, or to add two numbers together before raising to a power. The way to override the standard rules of 'multiplication before addition' and 'powers before multiplication', etc. is to use brackets.

> **Brackets in a calculation mean 'do this first'.**

So, in the calculation $(3 + 2) \times 4$, you should add the 3 and the 2 first (to give 5), then multiply by 4, i.e. $(3 + 2) \times 4 = 5 \times 4 = 20$. Similarly, in the calculation $(3 + 4)^2$, you should add the 3 and the 4 first before squaring. So $(3 + 4)^2 = 7^2 = 49$. Note that these give different answers from the calculations *without* brackets: for example, $3 + 2 \times 4 = 3 + 8 = 11$.

You can do calculations including brackets on a scientific calculator by using its brackets keys. Try the calculation $(3 + 2) \times 4$ on your calculator now, and make sure that you can get an answer of 20, not 11.

Strictly speaking, brackets are only needed to override the other rules of precedence, and they are not needed in calculations such as $3 + (2 \times 4)$. In the absence of the brackets, you or your calculator would follow the rule and do the multiplication first in any case. However, brackets are often used in calculations for clarity, even when they are not strictly necessary. For example, the calculation $6 \times 4 + 12 \times 5$ is more understandable and 'readable' if it is written as $(6 \times 4) + (12 \times 5)$, even though the brackets are not essential here. You are encouraged to write brackets in your calculations whenever they help you to express your working more clearly.

## 2.4 BEDMAS

Fortunately, there is an easy way to remember the correct order in which arithmetic operations should be carried out. The rules are neatly summed up in the acronym BEDMAS (or if it helps you to remember: **B**ig **E**lephants **D**estroy **M**ice **a**nd **S**nails). You may find the BEDMAS acronym helpful in your future studies and in other mathematical tasks that you encounter. Each letter of BEDMAS refers to one type of operation in a calculation:

**B** – brackets

**E** – exponents (also known as powers)

**DM** – divide or multiply (working from left to right)

**AS** – add or subtract (working from left to right)

Remember to always work from left to right as you use the BEDMAS order of operations. For example, to solve the calculation:

$(2 + 4)^2 - 6 \times 4 \div 8 + 7$

Step 1 is to first solve any parts of the calculation that are inside brackets (B):

$(2 + 4)^2 - 6 \times 4 \div 8 + 7$ becomes $6^2 - 6 \times 4 \div 8 + 7$

Step 2 is to solve any exponents (E) also known as powers. Remember that powers are shown as superscript numbers, in this case the superscript 2 in $6^2$. Remember that $6^2$ is 6 multiplied by itself ($6 \times 6$), which is 36:

$6^2 - 6 \times 4 \div 8 + 7$ becomes $36 - 6 \times 4 \div 8 + 7$

Step 3 is to do any divisions or multiplications (DM), always working from left to right:

$36 - 6 \times 4 \div 8 + 7$ becomes $36 - 24 \div 8 + 7$

$36 - 24 \div 8 + 7$ becomes $36 - 3 + 7$

Step 4 is to do any additions or subtractions (AS), always working from left to right:

$36 - 3 + 7$ becomes $33 + 7$

$33 + 7$ becomes 40, the final answer!

**Practice questions 3**

To practise the BEDMAS rules, try the following calculations.

(a) $150 \div 10 \times 3$

(b) $150 \div (10 \times 3)$

(c) $(2 \times 3)^2$

(d) $2 \times 3^2$

(e) $(10 - 5) \times (3 + 1) + 4$

(f) $(3 + 2)^2 - (7 - 4)^2$

# 2.5 Less than zero - calculating with negative numbers

A number above zero is referred to as a 'positive number' and can be shown either with a + sign in front or with no sign (Figure 1). A number below zero is referred to as 'negative number' and is always shown with a minus (−) sign in front. Notice from Figure 1 that the numbers get larger (or more positive) the further they are to the right, so +7 is larger than +3, and −4 is larger than −8.
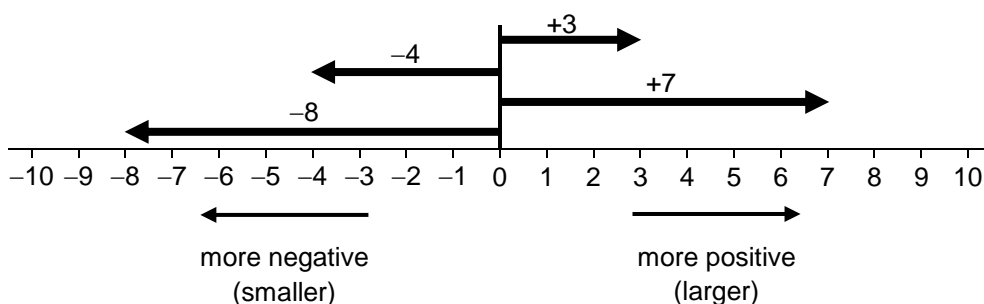


**Figure 1** A number line showing positive and negative numbers. (We have used the convention of showing positive numbers without a sign.)

### 2.5.1 Adding and subtracting negative numbers

You can use the number line in Figure 1 to help with simple addition and subtraction calculations. When you add a positive number you move further to the right on the number line, and when you subtract a positive number you move further to the left. For example:

$3 + 5 = 8$      (start at 3 and move 5 places to the right)

$-3 + 4 = 1$      (start at -3 and move 4 places to the right)

$6 - 8 = -2$      (start at 6 and move 8 places to the left)

$-1 - 6 = -7$      (start at −1 and move 6 places to the left)

You can also use the number line to add or subtract negative numbers. But in this case adding or subtracting a negative number goes in the *opposite* direction, if you add a negative number you move further to the *left* on the number line, and if you subtract a negative number you move further to the *right*. For example:

$5 + (-3) = 2$     (start at 5 and move 3 places to the *left*)

$-3 + (-5) = -8$   (start at −3 and move 5 places to the *left*)

$4 - (-3) = 7$     (start at 4 and move 3 places to the *right*)

$-6 - (-3) = -3$   (start at −6 and move 3 places to the *right*)

(the brackets are put in to these calculations to make it clearer).

You should be able to see from these examples that when it comes to adding and subtracting negative numbers the rules are:

(1)  Adding a negative number is the same as subtracting the corresponding positive number.

(2)  Subtracting a negative number is the same as adding the corresponding positive number.

Or you may find it easier to remember that when two signs appear next to each other and are different, then you subtract. When two signs are next to each other and they are the same, you add. For example:

$6 + (-8) = 6 - 8 = -2$

$2 - (-4) = 2 + 4 = 6$

## 2.5.2 Multiplying and dividing negative numbers

The rules for multiplying and dividing negative numbers are:

(1)  If you multiply or divide two numbers which have the *same* sign, the answer is positive.

(2)  If you multiply or divide two numbers which have *different* signs, the answer is negative.

For example:

- $2 \times (-3) = -6$
- $-4 \div 2 = -2$
- $-4 \div (-2) = 2$

**Practice questions 4**

Try the following calculations without using your calculator:

(a)  $3 - 6$

(b)  $-4 - (-2)$

(c)  $5 + (-1)$

(d)  $-7 + 4$

(e)  $-7 \times 5$

(f)  $4 \times (-3)$

(g)  $8 \div (-2)$

(h)  $-12 \div (-3)$

Now try them with your calculator (which ought to agree).

## 2.6 What your calculator doesn't tell you

Often the results of calculations need to include units of measurement in addition to numbers. However, calculators deal only with the numbers; so how do you decide what the unit of the answer to a calculation should be?

The answer comes from the fact that the unit must be the same on both sides of a mathematical equation. For example, if you want to add 3 litres and 5 litres, a calculator (or mental arithmetic) will tell you that 3 + 5 = 8. Then, since the unit must be the same on both sides of the equation, the answer must be 8 litres, i.e. 3 litres + 5 litres = 8 litres.

An important consequence of this requirement is that **you can't add or subtract quantities unless both quantities have the same unit**. You can't add 2 litres and 5 gallons; the total amount is neither 7 litres nor 7 gallons. To find the total amount, you need to convert 5 gallons into litres (or 2 litres into gallons) so that you are adding amounts measured in the same unit, usually a standard scientific unit called an SI unit – these are discussed in the next section.

> **Remember also that a calculator simply processes the numbers you key into it, so always consider carefully how to approach a calculation before using your calculator – and check the calculation at least once. It's very easy to 'miss-key' a number and end up with a wrong answer.**

# 3. Scientific units of measurement

The first requirement for measuring anything, such as length, mass, volume, temperature, and so on, is to decide on the unit of measurement. Scientists have agreed on an international standard set of units known as the SI system of units (from the French *Le Système Internationale d'unités*). The advantage of having a standard unit for each sort of measurement is that everyone uses it, so there is no need to convert laboriously from one unit to another to compare data in different countries. The SI system is built on seven 'base units', including the SI unit of length (distance), which is the metre, abbreviated to m. Table 1 shows all seven SI base units; note that you will only encounter the first four of them in SDK100.

**Table 1** The SI base units of measurement and their symbols.

| Quantity | Name | Symbol |
|---|---|---|
| length | metre | m |
| mass | kilogram | kg |
| time | second | s |
| amount of a chemical substance | mole | mol |
| electric current | ampere | A |
| thermodynamic temperature | kelvin | K |
| luminous intensity | candela | cd |

These seven base units are also the basis of more than twenty named 'derived' SI units. Table 2 shows only the derived SI units that you will encounter in SDK100. (SDK100 also introduces a few non-SI units which are still widely in use for historical reasons, for example, degrees Celsius or centigrade (ºC), food kilocalories (kcal) and also millimetres of mercury (mm Hg) which is used for blood pressure).

**Table 2** Some of the derived SI units of measurement used in SDK100 and their symbols.

| Quantity | Name | Symbol |
|---|---|---|
| temperature | celsius | °C |
| pressure | pascal | Pa |
| energy | joule | J |
| frequency | hertz | Hz |
| force | newton | N |
| voltage (electrical potential difference) | volt | V |
| absorbed dose (of ionising radiation) | gray | Gy |
| effective dose (of ionising radiation) | sievert | Sv |

The SI system has also established a set of prefixes that are placed in front of the unit names or symbols when specifying multiples and subdivision of the units, and these are explained in the next section.

## 3.1 Prefixes for multiples and subdivisions of scientific units

Using metres as an example, Table 3 lists the standard prefixes that can appear before all SI units of measurement to specify multiples or subdivisions of an SI unit.

**Table 3** Prefixes for SI units of measurement

| Prefix (symbol) | Multiple or subdivision of base units | Example in metres | Metres in powers of ten |
|---|---|---|---|
| tera (T) | 1 000 000 000 000 | 1 Tm = 1 000 000 000 000 m | $10^{12}$ m |
| giga (G) | 1 000 000 000 | 1 Gm = 1 000 000 000 m | $10^{9}$ m |
| mega (M) | 1 000 000 | 1 Mm = 1 000 000 m | $10^{6}$ m |
| kilo (k) | 1000 | 1 km = 1000 m | $10^{3}$ m |
| – | 1 | 1 m | $10^{0}$ m |
| deci (d) | 1/10th | 1 dm = 0.1 m | $10^{-1}$ m |
| centi (c) | 1/100th | 1 cm = 0.01 m | $10^{-2}$ m |
| milli (m) | 1/1000th | 1 mm = 0.001 m | $10^{-3}$ m |
| micro (µ) | 1/1 000 000 | 1 µm = 0.000 001m | $10^{-6}$ m |
| nano (n) | 1/1 000 000 000 | 1 nm = 0.000 000 001 m | $10^{-9}$ m |
| pico (p) | 1/1 000 000 000 000 | 1 pm = 0.000 000 000 001 m | $10^{-12}$ m |

So, for example, large distances can be measured in kilometres (km), and smaller distances or lengths can be measured in millimetres (mm). The prefix kilo means 'one thousand', so a kilometre is one thousand metres. The prefix milli means 'one-thousandth', so a millimetre is one-thousandth of a metre. Therefore:

1 km = 1000 m; 1 m = 1000 mm; 1 mm = $\frac{1}{1000}$ m.

- How many millimetres are there in 1 kilometre?

▫ 1 km = 1000 m and 1 m = 1000 mm, so 1 km = 1000 × 1000 mm = 1 000 000 mm, i.e. a million millimetres.

- What is the most appropriate unit prefix and symbol to use for the following: a thousand grams; a thousandth of a second?

▫ The most appropriate unit prefix and symbol are kg (kilogram) and ms (millisecond) respectively.

Another common prefix that you may have met is centi (as in centimetre, abbreviated to cm), which means 'one-hundredth', so 1 cm = $\frac{1}{100}$ m.

- How many centimetres are there in 25 metres?

▫ Since 1 m = 100 cm, 25 m = 25 × 100 cm = 2500 cm.

You may have been surprised to learn that the basic SI unit of mass is the kilogram not the gram. This is because, in general, the kilogram is of a more convenient size for measuring the masses of everyday objects, although you will have seen the number of grams quoted on, for example, packaged food. Just as the kilometre is equal to one thousand metres, so the kilogram is equal to one thousand grams, i.e. 1 kilogram = 1000 grams. Very small quantities are measured in milligrams, where one milligram (mg) is one-thousandth of a gram, so:

1kg = 1000g and 1mg = $\frac{1}{1000}$ g.

You will have noticed that Table 3 also shows multiples and subdivisions in another way – as powers of ten, which are explained in further in Section 6.

**Practice questions 5**

Carry out the following conversions, including the correct unit symbol in your answer.

(a) The number of joules in 5 kilojoules

(b) The number of milliseconds in 2 seconds

(c) The number of centimetres in 6.6 metres

(d) The number of micrograms in 2.2 grams

(e) The number of nanometres in 3 millimetres

(f) What is 250 grams in kilograms?

(g) What is 600 millimetres in metres?

(h) What is 3500 micrograms in grams?

## 3.2 Units of area and volume

Area is a measure of the size of an object's surface. For squares and rectangles, the area is found by multiplying the length by the width. So, if a rectangular garden pond has one side 4 m long and another side 2.5 m long, the numerical value of the area is given by multiplying 4 × 2.5, which is 10. What about the units? In this case, metres are being multiplied by metres (m × m) and, recalling from Section 2.2 the use of superscripts to indicate powers such as squares and cubes, m × m can be written as $m^2$. So the SI units of area are $m^2$, said as 'metres squared' or 'square metres'.

Volume is a measure of the space that a three-dimensional object occupies. The volume of a rectangular block is found by multiplying its length by its width by its height. A cube with 1 m long sides (i.e. 1 m × 1 m × 1 m) has a volume of 1 $m^3$. The standard unit of volume in the SI system is the cubic metre ($m^3$) (which can also be said as 'metre cubed').

However, volumes are also commonly measured in another unit, the litre (l) (Table 4). A litre is identical to a cubic decimetre ($dm^3$). A decimetre (dm) is 0.1 m or 10 cm, so a litre is equivalent to a cube with sides measuring 10 cm × 10 cm × 10 cm.

- How many litres are there in a cubic metre ($m^3$)?

□ Each side of a cubic metre (1 $m^3$) measures 10 decimetres (dm), so a cubic metre contains:
10 dm × 10 dm × 10 dm = 1000 $dm^3$ which is equivalent to 1000 litres.

For smaller volumes it's often more convenient to use the millilitre (ml) which is equivalent to a cubic centimetre ($cm^3$), or the even smaller unit, the microlitre (µl) which is equivalent to a cubic millimetre ($mm^3$).

**Table 4** Converting between litres and standard SI units of volume.

| Unit | Symbol | Equivalent standard SI unit |
|------|--------|-----------------------------|
| litre | l | $dm^3$ |
| millilitre | ml | $cm^3$ |
| microlitre | µl | $mm^3$ |

## 3.3 Compound units: speed and concentration

A compound unit is made up of two or more other units. Perhaps the most familiar example of the use of compound units is in the measurement of speed. Speed is the rate of travel, i.e. the distance travelled over one unit of time, such as a second, a minute or an hour. If a person ran 10 kilometres in 1 hour, then their speed would be 10 kilometres per 1 hour, written as 10 km $h^{-1}$. The superscript $^{-1}$ indicates that 10 km is divided by 1 hour (alternatively, a 'slash' is sometimes used to indicate the division, i.e. km/h).

If a car travelled a total of 120 kilometres in 3 hours its average speed can calculated by dividing the distance it has travelled by the total time, in this case:

120 km ÷ 3 h = 40 km $h^{-1}$

Note that what we are measuring is an *average* speed over the time period. The answer would be the same if the car travelled at a steady 40 km $h^{-1}$ or slowed down at some points and went faster in between.

> **By convention, in compound units there is a space between each unit to avoid confusion between, for example, metres per second (m $s^{-1}$) and a millisecond (ms), which is a thousandth of a second.**

Another example of the common use of compound units is chemical concentration, which is the amount of a particular substance present in a volume of solution, or a volume of space. For example the concentration of sugar dissolved in a cup of tea might, for example, be measured in grams per litre, written as g $l^{-1}$.

In Topic 2 you will calculate the energy expended by the body during different types of physical activity using a value called a Metabolic Equivalent, or MET, which combines three units: energy, mass and time (kcal $kg^{-1}$ $h^{-1}$) - see Section 8.1.3 of this workbook.

**Practice questions 6**

Try the following calculations:

(a) If a nerve signal travels 0.1 metre in 1 millisecond, what is its speed in m $s^{-1}$?

(b) The labelling of a standard cola drink indicates that it contains 10.6 g of sugar per 100ml, while a lemonade drink contains 22.5g of sugar per 250 ml. What is the concentration of sugar in g $l^{-1}$ in each type of drink?

# 4. Fractions, percentages and ratios

Fractions, percentages and ratios are all ways of expressing proportions, i.e. they show the relationship between two or more numbers.

## 4.1 Fractions

A fraction is a part of a whole and is the result of dividing the whole amount into equal parts. If, for example, you divide a cake into four parts, one of those parts will be one quarter, which can be written as the fraction $\frac{1}{4}$. If you eat that quarter, what you will have left is three quarters, or $\frac{3}{4}$, of the cake.

Fractions are characterised by a *numerator* (the number on top) and a *denominator* (the number on the bottom). Note also that fractions can be written in two ways for example $\frac{1}{4}$ or 1/4. The latter is sometimes more convenient in a line of text.

Fractions are usually written in the simplest form with the lowest possible whole numbers in the numerator and the denominator. For example, in $\frac{4}{6}$ the numerator and denominator can both be divided by 2 to give $\frac{2}{3}$.

**An important rule to remember is that the value of a fraction is unchanged if its numerator and denominator are both multiplied by the same number, or both divided by the same number.**

You can often find the simplest form of a fraction by dividing the top and bottom by simple numbers in several easy steps until you reach the point where you can no longer divide, for example:

$\frac{300}{2000}$ if you divide both the top and the bottom by 100 becomes $\frac{3}{20}$

and $\frac{450}{1050}$ divided top and bottom by 10 is $\frac{45}{105}$, then divided by 5 is $\frac{9}{21}$ then by 3 is $\frac{3}{7}$.

## 4.2 Percentages

You have probably met percentages in various contexts, for example a 3% pay rise or a 20% discount on goods in a sale. A percentage is a fraction expressed in hundredths. So, $\frac{1}{2}$ or one-half, is $\frac{50}{100}$, or fifty-hundredths, and this can be expressed as 50 per cent, which is usually written as 50%. This literally means 50 in every 100.

The advantage of using percentages is that you are always dealing with hundredths, so percentages are easy to compare, whereas fractions can express different numbers of parts for example, eighths, sixteenths or fiftieths, which are difficult to compare. It is not immediately obvious that $\frac{19}{25}$ is larger than $\frac{15}{20}$ but if these fractions are expressed as percentages, i.e. 76% and 75%, respectively, it is easy to see that the former number is the larger of the two. But how are fractions converted into percentages, or percentages into fractions?

Converting a fraction to a percentage follows a very simple rule: you multiply the fraction by 100% – that's all there is to it. For example:

$\frac{1}{2} \times 100\% = 1 \div 2 \times 100 = 50\%$

(To do this using a calculator, you would key $1 \div 2 = 0.5$, then $0.5 \times 100 = 50\%$)

$\frac{3}{4} \times 100\% = 3 \div 4 \times 100\% = 75\%$

To convert a percentage into a fraction, you need to remember that a percentage is a fraction expressed in hundredths and then cancel as appropriate by dividing the top and bottom by a simple number, so:

$75\% = \frac{75}{100} = \frac{15}{20} = \frac{3}{4}$

$$15\% = \frac{15}{100} = \frac{3}{20}$$

## 4.3 Calculating fractions and percentages of numbers

You can use the methods described above to work out the relationship between any two numbers as a fraction or a percentage. For example, if the total number of deaths worldwide from all causes in 2012 was 55 859 000 and the total number of deaths from coronary heart disease was 7 356 000, what percentage of all deaths worldwide were from coronary heart disease?

The first step is to write down the fraction of deaths that were from coronary heart disease, i.e. 7 356 000 of 55 859 000:

$$\frac{7356000}{55859000}$$

The calculation to convert this fraction into a percentage below should then look familiar, even though the numbers in the fraction are very large:

$$\frac{7356000}{55859000} \times 100\%$$

You will need your calculator to work this out, so key in $7356000 \div 55859000 \times 100\% = 13\%$ (to the nearest whole number)

- What percentage of 26 000 is 6500?
- Write down the fraction first, then multiply it by 100%:

$$\frac{6500}{26000} \times 100\% = 25\%$$

But how would you work out what a certain fraction or percentage of a given number is? For example, what is $\frac{3}{4}$ of 12? First of all 12 can be divided into four equal parts or quarters ($12 \div 4 = 3$). Then, since you want three-quarters, which is three times as big, you multiply one of these parts by three ($3 \times 3 = 9$). This calculation can be written as:

$\frac{3}{4} \times 12$ which is the same as $\frac{3 \times 12}{4} = \frac{36}{4} = 9$

So '$\frac{3}{4}$ of' a number simply means multiply $\frac{3}{4}$ by that number.

- What is $\frac{2}{3}$ of 18?
- $\frac{2}{3}$ of 18 means $\frac{2}{3} \times 18$, which is 12.

Working out 75% of a number can be done in a similar way if you remember that $75\% = \frac{75}{100}$.

So, 75% of 40 is $\frac{75}{100} \times 40 = 30$

- If a bottle of wine contains a volume of 750 millilitres (ml) and the wine is 12% pure alcohol by volume, how many millilitres of pure alcohol are there in the bottle?
- To answer the question you need to work out 12% of the total volume:

$\frac{12}{100} \times 750 = 90$ ml of pure alcohol.

Some modern calculators will convert directly between fractions and percentages and find the percentage of a number at the press of a button. However, it is worth making sure that you understand the meaning of fractions and percentages before letting your calculator do the work for you. This will enable you to check that the answer you obtain is reasonable: for example, 48% of a quantity should be just less than half of it.

Calculate the following:

(a) $\frac{2}{5}$ of 20

(b) $\frac{7}{8}$ of 24

(c) 15% of 60

(d) 60% of 5500

(e) 3 as a percentage of 600

(f) 12 166 as a percentage of 55 300

(g) A glass of beer contains a volume of 500 millilitres (ml). If the beer is 5% pure alcohol by volume, how many millilitres of pure alcohol are there in the glass?

# 4.4 Ratios

Another way of expressing the proportion of one quantity to another is to consider the ratio of the two numbers. Suppose that a class of 25 children has 15 girls and 10 boys. The ratio of girls to boys is 15 to 10, which is usually written as 15:10, i.e. with a colon (:) between the two numbers. This can be simplified to 3:2 (by dividing both 15 and 10 by 5, in the same way as fractions can be simplified by dividing the numerator and denominator by the same whole number). Ratios are usually expressed in the simplest possible whole numbers and should not include units.

- If average daily water use in the UK for baths and showers is 50 litres per person per day, compared with 12 litres per day for dish washing. What is the ratio of water use for baths and showers to that for dish washing? You should express the ratio in its simplest possible form.

- Since both quantities are expressed in 'litres per person per day', you can ignore the units in the ratio and express it in simple numbers as 50:12. Dividing both numbers by two allows the ratio to be simplified to 25:6.

You need to take care when asked to give the ratio of two numbers, as the following example shows.

- Suppose 2 out of 10 people in the UK drink bottled water. What is the ratio of people who drink bottled water to those who don't?

- The ratio is 2:8, which can be expressed as 1:4 (by dividing both numbers by two).

Did you fall into the trap and answer 2:10 or 1:5? This is the ratio of people who drink bottled water to the *total* number of people. Of course, if you had been asked what is the ratio of people who don't drink bottled water to those who do the answer would have been 8:2 or 4:1. So, always read the question carefully.

Ratios are particularly useful where the relative proportions of two or more parts of a whole are being considered. For example, the ratio of males to females in the general population of the UK is about 1:1. Ratios can be used to compare more than two quantities, so if you use 150 grams of flour, 150 grams of sugar and 100 grams of fat in making a cake, these quantities are in the ratio 150:150:100, which simplifies to 15:15:10 and then to 3:3:2. However, this module, you will only be asked to consider ratios of a quantity to one other quantity.

In a group of Open University students, 16 were men and 8 were women.

(a) What was the ratio of women to men in the group?

(b) What fraction of the total group were women?

# 5. Decimal numbers

Decimal numbers use the ten digits, 0 to 9. Decimal numbers are used throughout science, and you need to become proficient at adding, subtracting, multiplying and dividing them. Fortunately, your calculator will take the pain out of the calculations, so you can concentrate on understanding what the numbers mean.

The position of each digit in a number indicates its value. Table 5 is a decimal place–value table for the number 13 259 147 (thirteen million two hundred and fifty nine thousand one hundred and forty seven. Note that putting either a space (as above) or a comma (,) between every third digit (counting back from the end of the number) makes it easier to see which digits relate to the millions, which to the hundred thousands, and so on. In the decimal place-value table, the value of each column is ten times the value of the column to its right.

**Table 5** A decimal place–value table for the number 13 259 147.

| Ten millions | Millions | Hundred thousands | Ten thousands | Thousands | Hundreds | Tens | Units |
|---|---|---|---|---|---|---|---|
| 10 000 000 | 1 000 000 | 100 000 | 10 000 | 1000 | 100 | 10 | 1 |
| 1 | 3 | 2 | 5 | 9 | 1 | 4 | 7 |

A fraction of a whole number can also be written as a decimal number. For example one-half $\frac{1}{2}$ can also be written as the decimal number 0.5, so $\frac{1}{2}$, 50% and 0.5 all have the same meaning. If a decimal number includes a 'point' (full stop) the digits after the decimal point represent a fraction of one unit. The first digit after the point refers to 'tenths' of one unit; the second digit after the point refers to 'hundredths' of one unit; the third digit after the point refers to 'thousandths' of one unit; and so on.

Table 6 is a place-value table for the decimal number 2.5379 spoken as 'two point five three seven nine'. Note that the part of the number before the decimal point is spoken as a whole number, and the part after the point is spoken as a series of individual digits.

> **Beware, because some countries (and some calculators) use a comma (,) instead of a decimal point. The same countries (and this includes continental Europe) use a full stop, instead of a comma, to indicate thousands.**

**Table 6** A decimal place–value table for the number 2.5379.

| Units | Tenths | Hundredths | Thousandths | Ten-thousandths |
|---|---|---|---|---|
| 1 | 0.1 | 0.01 | 0.001 | 0.0001 |
| 2 | 5 | 3 | 7 | 9 |

The part of the number before the decimal point represents a whole number, and the part after the decimal point represents the fraction, something between zero and one, that has to be added on to the whole number. Note that when there is no whole number, i.e. a number that is less than one, such as 0.5, it is usual to print or write a zero in front of the decimal point otherwise the decimal point might be overlooked. (Your calculator, however, may not always show the zero.)

Conversion of any fraction to a decimal number is straightforward with a calculator. All you have to do is divide the number on the top of the fraction by the number on the bottom. Try this for yourself: with the fraction $\frac{15}{40}$. You should obtain the decimal number 0.375.

## 5.1 Rounding decimal numbers

The number of digits after the decimal point in a number is termed the 'number of decimal places'. So, for example, 0.745 is expressed to three decimal places, 7 is in the first decimal place, 4 is in the second decimal place and 5 is in the third decimal place.

- To how many places of decimals is 86.587 156 83 expressed?

- Eight; there are eight digits after the decimal point.

Often when you do a calculation your calculator will display an answer with perhaps seven or even nine decimal places: for example, it may indicate that $\frac{1}{3}$ is 0.333 333 333 (recurring). In most cases, it is not necessary to give all of these digits. It is probably sufficient to know that $\frac{1}{3}$ is about 0.33, and to forget about the thousandths, the ten-thousandths, and so on. Sometimes it is enough to know that $\frac{1}{3}$ is about 0.3. When you approximate in this way, you are 'rounding' the number. However, rounding is a little more complicated than just chopping off the unwanted digits.

If you want to round 1.2645 to two decimal places, you need to look at the first digit to be removed – in this case, 4. If the first digit removed is 0, 1, 2, 3 or 4, the last remaining digit – in this case, 6 – is left unchanged. So the answer is 1.26.

However, if the first digit to be removed is 5, 6, 7, 8 or 9, the last remaining digit is increased by one, a process described as 'rounding up'. So, for example, if 1.2645 is rounded to one decimal place, the answer is 1.3 – the first digit removed is 6, so the 2 is rounded up to 3.

The reason for rounding up when the first digit removed is 5 or greater is clear if you bear in mind that the number that is midway between 1.2 and 1.3 is 1.25. So, all numbers between 1.25 and 1.3 are closer to 1.3 than they are to 1.2. Therefore, it makes sense to round up the last remaining digit whenever it has been followed by a digit between 5 and 9. Note that, by convention, the digit 5 is usually rounded up.

- Round 86.587 156 83 to the nearest whole number.

- The first digit to be removed is the 5 immediately after the decimal point thus the number is rounded up to 87.

Sometimes, rounding a decimal number will produce a zero as the final digit; for example, both 1.803 and 1.798 become 1.80 when rounded to two decimal places. Don't be tempted to ignore the final zero in these cases though, because it contains important information about the decimal number. Quoting a length as 1.80 metres tells you that the measurement is between 1.7950 metres and 1.8049 metres because numbers within this range are equal to 1.80 metres when rounded to two decimal places. On the other hand, quoting the length as 1.8 metres means that it is between 1.750 metres and 1.849 metres, which is a much larger range.

It is often useful to 'round' a number up or down to the nearest decimal equivalent to make it more manageable and more easily understood. This is what we did in Topic 1 Section 4.1.1 (Activity 4.1) when calculating the percentage of all deaths in 2012 that were due to infectious diseases.

9 923 000 ÷ 55 859 000 × 100% = 17.76437%, rounded up to 18%

We also rounded up when we said that 'almost 10 million' people died in 2012 from an infectious disease. In fact, the number was 9 922 975 (nine million, nine hundred and twenty-two thousand, nine hundred and seventy-five) deaths. We could have quoted a slightly more accurate figure than 'almost 10 million' if we had given the total as '9.9 million', which is expressing it to one decimal place, i.e. there is one digit after the decimal point.

Rounding reduces the accuracy of a number but provides an approximation that makes the data easier and quicker to understand. How much to round up depends on the circumstances, and factors such as the original accuracy of the number and the purpose for using it. One way of making this decision during a calculation is to round to a certain number of significant figures, which is explained in Section 7.

**Practice questions 10**

Round each of the following numbers to the nearest whole number, to one decimal place, to two decimal places, and to three decimal places. (a) 1.2648 (b) 0.82551 (c) 21.9184

# 6. Powers of ten and scientific notation

Rounding is one way of avoiding working with cumbersome numbers but another way of expressing very large or small numbers in a more convenient form is called powers of ten.

## 6.1 Powers of ten for very large numbers

When dealing with large numbers such as one thousand four hundred and sixty million (1 460 000 000), it is tedious to write all of those zeros and very easy to lose some or add extra ones by mistake. Fortunately, large numbers can be referred to without having to write out all of the zeros.

You may already know that 100 can be written as $10^2$ or 'ten squared' because a number multiplied by itself is said to be 'squared' (Section 2.2) and $10 \times 10 = 100$. If you write 100 as $10^2$, the ten (10) is known as the base number and the superscript two ($^2$) is called the power (or exponent). So this can also be referred to as 'ten to the power two':

$$10^2 \quad \text{\textit{power}}$$
$$\text{\textit{base number}}$$

This way of writing multiples of ten is therefore known as 'powers of ten'. Exactly the same process applies to converting even larger numbers to their equivalent power of ten. You can work out the powers by counting the zeros, e.g. 1 million is 1 000 000 or $10^6$ ('ten to the power six') and 1 billion is 1 000 000 000 or $10^9$ ('ten to the power nine').

The power of ten actually shows how many times 1 has been multiplied by 10. Taking $1 \times 10^3$ as an example, 1000 is $1 \times 10 \times 10 \times 10$. One thousand is therefore often written not just as $10^3$ but as $1 \times 10^3$ (one times ten to the power three' or just 'one times ten to the three'). Likewise, one million is either $10^6$ or $1 \times 10^6$.

Suppose that you wanted to write 2 million (2 000 000) as a power of ten. You would write it as $2 \times 10^6$, which is said aloud as 'two times ten to the power six' or more simply 'two times ten to the six'. The important thing to note here is the convention, known as *scientific notation*, which limits the size of the multiplier (the number before the multiplication sign) in a power of ten.

The rule of scientific notation is that the multiplier in a power of ten has to be *greater than or equal to 1 and less than 10*.

So, for example, 20 million is never written as $20 \times 10^6$ because this breaks the rule: the multiplier has to be less than 10. We can make $20 \times 10^6$ conform to scientific notation by writing it as $2 \times 10^7$ ('two times ten to the seven'). We did this by dividing the number 20 by 10 (to give 2) and then multiplying $10^6$ by 10 (to give $10^7$). If you are unsure why $10^6 \times 10 = 10^7$, remember that $10^6$ represents $10 \times 10 \times 10 \times 10 \times 10 \times 10$ (i.e. 10 multiplied by 10 six times), so multiplying it by 10 one more time gives $10^7$.

Using the powers of ten notation, the total number of deaths worldwide from all causes in 2012, which is 55 859 000, could be written as $5.5859 \times 10^7$. The '7' in $10^7$ tells you how many times 5.5859 has been multiplied by 10 to give the final number of 55 859 000. It is seven times, i.e. the number comprises:

$5.5859 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

Alternatively, you can think of each increase by one in the power of ten as moving the decimal point one place to the left. To write 55 859 000 using scientific notation, you would need to move the decimal place seven places to the left, so each time you move the decimal point adds a power of ten:

5 . 5 8 5 9 0 0 0  $= 5.5859 \times 10^7$

This final number would be spoken as 'five point five eight five nine times ten to the seven'.

**Practice questions 11**

Express the following numbers in scientific notation.

(a) 100 000 000

(b) 35 000

(c) $95 \times 10^5$

(d) $0.51 \times 10^3$

Convert the following scientific notation numbers into decimal form:

(e) $7.3 \times 10^4$

(f) $4.44 \times 10^5$

(g) $6.05 \times 10^3$

## 6.2 Powers of ten for very small numbers

The powers of ten notation can also be used for small numbers, such as 0.000 000 0002 m.

Write down the next two numbers in each of the following two sequences.

| 10 000 | 1000 | 100 | ? | ? |
|---|---|---|---|---|
| $1 \times 10^4$ | $1 \times 10^3$ | $1 \times 10^2$ | ? | ? |

In the first sequence, each successive number is divided by 10 (i.e. one zero is taken off the end) so the number that follows 100 is 10 and the number that follows 10 is 1. In the second sequence of numbers, each successive number has 1 subtracted from its power, so the first answer is $1 \times 10^1$ because $2 - 1 = 1$. For the second answer, you must subtract 1 from the power 1 to get $1 \times 10^0$.

In fact, both sequences are the same: 10 000 is $1 \times 10^4$, 1000 is $1 \times 10^3$, 100 is $1 \times 10^2$, and 10 is $1 \times 10^1$. The implication is that $1 = 1 \times 10^0$ and hence $10^0 = 1$. This makes perfectly good sense if you recall that, in the second sequence given above, the power is the number of times that 1 is multiplied by 10 (e.g. $10^2 = 1 \times 10 \times 10$). For $1 \times 10^0$, 1 is multiplied by 10 no times at all, leaving it as 1.

Using the same rules, write down the next number in each of the following sequences.

| 100 | 10 | 1 | ? |
|---|---|---|---|
| $1 \times 10^2$ | $1 \times 10^1$ | $1 \times 10^0$ | ? |

In the first sequence dividing 1 by 10 gives $\frac{1}{10}$ or 0.1 as the next number. But what about the second sequence? The answer is more straightforward than it may seem. You simply continue to subtract 1 from the powers of ten so that the next number in the sequence has a negative power of ten ($1 \times 10^{-1}$) because $0 - 1 = -1$. The two sequences are equivalent, so $1 \times 10^{-1} = 0.1$. Just as a positive power of ten denotes how many times a number is *multiplied* by 10, so a negative power of ten denotes how many times a number is *divided* by 10.

- What is the meaning of $10^{-2}$?
  - The power is −2, so you must divide 1 by 10 twice. That is, $1 \div 10 \div 10 = 1 \div 100 = 0.01$

For example, suppose a raindrop has a width of about 0.002 m. This distance could be given in scientific notation as $2 \times 10^{-3}$ m:

$0.002 = 2 \div 10 \div 10 \div 10 = 2 \times 10^{-3}$.

Alternatively, you can think of each increase by one in the negative power of ten as moving the decimal point one place to the right. To write 0.002 using scientific notation, you would need to move the decimal place three places to the left. Each time you move the decimal point adds a negative power of ten:

$0 \ . \ 0 \ 0 \ 2 \ = 2 \times 10^{-3}$

This final number would be spoken as 'two times ten to the minus three'.

Recall from Section 6.1 that, when writing large numbers in scientific notation, the power of ten should be accompanied by a multiplier that is *greater than or equal to 1 and less than 10*. The same convention is used when dealing with small numbers and hence negative powers of ten. This is why 0.002 m, the width of the raindrop, is given in scientific notation as $2 \times 10^{-3}$ m, and not as $0.2 \times 10^{-2}$ m or $20 \times 10^{-4}$ m.

**Practice questions 12**

Write out the following numbers using scientific notation

(a) 0.00354

(b) 0.08274

(c) 0.00000224

Convert the following scientific notation numbers into decimal form.

(d) $5.64 \times 10^{-3}$

(e) $1.673 \times 10^{-5}$

(f) $2.3387 \times 10^{-2}$

# 6.3 Calculations using powers of ten

## 6.3.1 Multiplying or dividing powers of ten

You will come across the need to divide or multiply powers of ten in Topic 8 of SDK100. These processes have a quite simple rule:

**When multiplying you add the powers, and when dividing you subtract the powers.**

For example: $(1 \times 10^3) \times (1 \times 10^2) = 10^3 \times 10^2 = 10^{3+2} = 10^5$

(You can try this out the long way to prove it works: $10^3 \times 10^2 = 1000 \times 100 = 100\,000 = 10^5$).

Similarly, $10^3 \div 10^2 = 10^{3-2} = 10$ (or doing it the long way, $1000 \div 100 = 10$)

A calculation like $(2 \times 10^2) \times (3 \times 10^4)$ requires two steps:

Step 1 is to multiply just the numbers: $2 \times 3 = 6$

Step 2 is to multiply just the powers of ten by adding the powers: $10^2 \times 10^4 = 10^6$

So the final answer is $= 6 \times 10^6$

To carry out the calculation $(4 \times 10^{-3}) \div (2 \times 10^2)$

Step 1 is to divide just the numbers: $4 \div 2 = 2$

Step 2 is to divide just the powers of ten by subtracting the powers: $10^{-3} \div 10^2 = 10^{-3-2} = 10^{-5}$ (remember that $-3-2 = -5$, Section 2.5.1)

So the final answer is $2 \times 10^{-5}$

## 6.3.2 Adding and subtracting powers of ten

Adding and subtracting powers of ten is a little trickier and luckily you won't need to do these types of calculation in SDK100, but for completeness it is useful to remember that:

You can only add and subtract 'like terms' that have the same base and power

Since all numbers in scientific notation have the same base (10), we need only worry about the power. To add or subtract, two numbers in scientific notation, one of them must be manipulated so that they both have the same power. There are various ways of doing this, but for example to add $(1 \times 10^2)$ and $(1 \times 10^4)$ it's best to start by changing the second number to a ten to the power of two. In other words, $1 \times 10^4$ is the same as $0.01 \times 10^2$, so:

$(1 \times 10^2) + (1 \times 10^4)$ can be changed to $(1 \times 10^2) + (100 \times 10^2)$.

You can then simply add the 1 to the 100 to give:

$(1 \times 10^2) + (100 \times 10^2) = 101 \times 10^2 = 1.01 \times 10^4$

Similarly, to subtract $(1 \times 10^2)$ from $(1 \times 10^3)$,

$(1 \times 10^3) - (1 \times 10^2)$ can be changed to $(10 \times 10^2) - (1 \times 10^2) = 9 \times 10^2$

Again, you can try converting all these numbers to standard 'long' numbers to check that this method works.

**Practice questions 13**

Calculate the following giving your answer in scientific notation:

(a)  $(1 \times 10^4) \times (1 \times 10^3)$

(b)  $10^2 \times 10^5$

(c)  $(2.5 \times 10^3) \times (4 \times 10^2)$

(d)  $(1 \times 10^6) \div (1 \times 10^3)$

(e)  $10^8 \div 10^6$

(f)   $(3.5 \times 10^8) \div (5 \times 10^3)$

## 6.3.3 Using a calculator for scientific notation

If you have the right sort of scientific calculator you can enter values using scientific notation. Use the special button provided for entering scientific notation (check your calculator instructions). This button might be labelled as EXP, EE, E or EX, but there is considerable variation between calculators. Make sure that you can find the appropriate button on your calculator. Using this sort of button is equivalent to typing the whole of '× 10 to the power'. So, on a particular calculator, keying 2.5 EXP 12 enters all of $2.5 \times 10^{12}$.

To enter a number such as $10^9$ on your calculator using the scientific notation button, it is helpful to remember that $10^9$ is written as $1 \times 10^9$ in scientific notation, so you will need to key in for example 1 EXP 9. In addition to being able to enter numbers in scientific notation on your calculator, it is important that you can understand your calculator display when it gives an answer in scientific notation. If you have a scientific calculator, enter the number $2.5 \times 10^{12}$ on your calculator and look at the display. Again there is considerable variation among calculators, and you may see any of the following:

  $2.5 \times 10^{12}$          $2.5_E12$                  $2.5e+12$               $2.5\ 12$                  $2.5\ ^{12}$

The 12 at the right of the display is always the power of ten, but note that the ten itself is frequently not displayed. If your calculator displays 2.5 $^{12}$ (the last example in the list above), you will need to take particular care; this does not mean $2.5^{12}$ on this occasion. You should be careful not to copy down a number displayed in this way on your calculator as an answer to a question; this could cause confusion at a later stage. No matter how scientific notation is entered and displayed on your calculator or computer, when writing it on paper you should always use the form exemplified by $2.5 \times 10^{12}$.

> **To enter a number such as $5 \times 10^{-6}$ on your calculator, you may need to use the button labelled something like +/− or ± in order to enter the negative power.**

# 7. Significant figures

Calculations using scientific measurements often result in an answer with a large number of decimal places, particularly if you use a calculator. You then need to decide how many 'significant figures' to quote in the final answer. Significant figures really just mean the most important digits – the ones you are most certain of.

In straightforward cases, the number of significant figures in a number is found simply from counting the number of digits. In the number 23 509, the 2 is the most significant digit, because it tells us that the number is twenty thousand and something. 3 is the next most significant and then 5. If, as in this case, a zero appears between two significant digits, it is also counted as a significant digit. The last significant digit here is 9, so the number 23 509 has five significant figures.

For the number 0.000324, the zeros at the beginning act only as place holders (they are there to place the decimal point in the correct location) so the 3 is the most significant digit, because it tells us that the number is 3 ten thousandths and something. The 2 is the next most significant and then the 4. So the number 0.000324 has three significant figures.

> Note that 'trailing zeros' (i.e. zeros at the end of a number) are only considered significant if they are at the end of a number with a decimal point in it. For example, the number 95 has two significant figures (9 and 5) but the number 95.00 has four (9, 5, zero, zero). This convention relates to the investigator's confidence in the precision of a measurement – a measurement of 95.00 metres is claiming to be more precise than one of 95 metres.  The number 0.0500 has three significant figures (5, zero and zero).
>
> Trailing zeros at the end of a whole number however are *not* considered significant (because they act as placeholder). For example the number 25 000 only has two significant figures, 2 and 5. This can sometimes lead to confusion about whether the number means exactly 25 000, or an approximation of 'twenty five thousand and something' to two significant figures. If you wanted to express the exact number 25 000 to, for example, three significant figures it's best to turn it into a power of ten: $2.50 \times 10^4$ so that it's clear what you mean. You probably won't need to use this rule in SDK100, but it's useful to be aware of it.

The rules for rounding significant figures are the same as usual (Section 5.1). If the next number is 5 or more you should round up but if it is less than 5 you should not round up. So, the number 58 567 expressed to three significant figures would be 58600, or as a power of ten, $5.86 \times 10^4$.

As a rule of thumb, when multiplying and dividing numbers, the number of significant figures in the result of a calculation should be the same as the value with the fewest significant figures used in the calculation. For example, if the length of a flower bed is 4.5 m and its width is 1.09 m, then the calculated area is 4.5 m $\times$ 1.09 m = 4.905 m$^2$ (Section 3.2). However, since the length of 4.5 m is measured only to two significant figures (4 and 5), you would only be justified in quoting the area to two significant figures, so we would round the calculated value to 4.9 m$^2$.

> A scientific calculator can be set up to work to a particular number of significant figures in scientific notation mode. Calculators differ in how this is set and you will need to consult the instruction booklet for your calculator to find out exactly how to do it but, for example, it might be something like MODE – SCI – 4 to choose to work in scientific notation to 4 significant figures. Even if you choose to work in, say, 4 significant figures, the calculator will still carry out the calculation using the full number and correcting at the end so no data will be lost.

# 8. Working with equations

A mathematical equation is a statement that two things are equal. An equation consists of two expressions, one on either side of an equals sign, for example:

$6 + 8 = 14$

Equations are a very useful way of calculating an unknown value if you know the other values in the equation. For example, if we let the letter $x$ represent an unknown value;

$x = 4 + 3$

You should easily be able to work out that the value of $x$ is 7. The process of working out the unknown value is known as 'solving' the equation. Letters or symbols are often used for the unknown variables in a special type of equation (sometimes known as a formula) that represents the general relationship between two or more variables.

**The use of letters or symbols to represent the variables in an equation is referred to as algebra. Variables are always written in italics.**

For example, the volume of a box is represented by the algebraic formula:

$v = l \times h \times w$   or more simply $v = l\,h\,w$

where $v$ is volume, $l$ is length, $h$ is height and $w$ is width. So if the length of a box is 10 cm, the height is 5 cm and the width is 5 cm, the volume of the box would be:

$v = 10 \times 5 \times 5 = 250 \text{ cm}^3$

In this case, volume ($v$) is the 'subject' of the formula, the single variable (usually placed on the left of an equation) which is equal to the expression on the other side of the equals sign. One of the useful things that algebra can help you do is 'rearrange' a formula so that another variable becomes the subject. If, for example, you wanted to know the width of a box instead of the area, you could rearrange the original equation:

$v = l \times h \times w$

First, divide both sides by height, $h$ (on the right side of the equation $h$ is therefore 'cancelled out' when it is divided by itself, so it disappears):

$$\frac{v}{h} = l \times w$$

Then divide both sides by length, $l$ (again this cancels out $l$ on the right side):

$$\frac{v}{h \times l} = w$$

Now you can swap the equation round so that the new subject, $w$, is on the left:

$w = \dfrac{v}{h \times l}$ or more simply w $= \dfrac{v}{h\ l}$

To rearrange any equation, you may:

- add or subtract the same quantity to or from both sides
- multiply or divide both sides by the same quantity

It is always best to rearrange the equation in algebraic from before substituting numbers. The key thing to remember is that whatever you do to one side of the equation you must also do to the other side.

# 8.1 Equations, formulas and calculations used in SDK100

In SDK100 you will use several simple equations that combine different maths skills.

## 8.1.1 Body mass index (BMI)

In Topic 2, Section 1.3.2 you will use a method of classifying body composition called body mass index (BMI) which can be calculated from an individual's height and body mass using the equation:

$BMI = \dfrac{\text{mass (kg)}}{\text{height (m)}^2}$

A number squared is a number multiplied by itself, so:

$BMI = \dfrac{\text{mass (kg)}}{\text{height (m)} \times \text{height (m)}}$

For example, the BMI of a woman who is 1.7 metres tall with a body mass of 79 kg is calculated as follows:

$BMI = \dfrac{79 \text{ kg}}{1.7 \text{ m} \times 1.7 \text{ m}} = \dfrac{79 \text{ kg}}{2.89 \text{ m}^2} = 27.3356 \text{ kg/m}^2$

However, as both values used in the calculation have only two significant figures (Section 7) the final answer should be rounded to 27 kg/m$^2$.

## 8.1.2 Basal metabolic rate (BMR)

In Topic 2 Section 5.1.4 you will use equations to estimate basal metabolic rate (BMR) - the energy expended by the body's metabolism:

- BMR for men (kilocalories per day) = (10 × mass in kg) + (6.25 × height in cm) − (5 × age in years) + 5
- BMR for women (kilocalories per day) = (10 × mass in kg) + (6.25 × height in cm) − (5 × age in years) − 161

To calculate the BMR of a 50 year-old woman who is 1.7 metres tall with a body mass of 79 kg, first convert her height from metres to centimetres. There are 100 cm in a metre (Section 3.1) so her height is 170 cm.

Recall also that in calculations, the rules of precedence (BEDMAS, Section 2.4) require that the parts of an equation inside brackets should be solved first:

BMR for women (kilocalories per day) = (10 × 79) + (6.25 × 170) − (5 × 50) − 161

BMR (kilocalories per day) = 790 + 1062.5 − 250 −161

So BMR = 1441.5 kilocalories per day

### 8.1.3 MET values

In Topic 2 Section 5.1.5 you will use a value called a Metabolic Equivalent (MET) to estimate the energy used during different sorts of physical activity. MET values are multiples of an individual's basal metabolic rate (BMR). When you are lying down and resting, you are expending only the energy required for metabolism which has a value of 1 MET (in other words, 1 MET is one times BMR). Walking at 4 km an hour (km h$^{-1}$) on a level surface has a value of 3 METs (i.e. three times BMR) while jogging has a value of 7 METs (Topic 1 Table 5.4).

For most people, 1 MET is equivalent to about 1 kilocalorie per kilogram of body mass per hour of exercise (1 kcal kg$^{-1}$ h$^{-1}$). A person with a body mass of 65 kg who walked at 4 km an hour (km h$^{-1}$) on a level surface (using the equivalent of 3 METs) for 2 hours would expend:

3 kcal kg$^{-1}$ h$^{-1}$ × 65 kg × 2 hours = 390 kcal in total

If the same person ate a piece of cake that provided 300 kcal of energy; to work out how long it would take them to use up that amount of energy by walking:

300 kcal = 3 kcal kg$^{-1}$ h$^{-1}$ × 65 kg × unknown number of hours ($h$)

To rearrange the equation you can divide both sides by 3 kcal kg$^{-1}$ h$^{-1}$ and 65 kg:

$$\frac{300 \text{ kcal}}{3 \text{ kcal kg}^{-1} \text{ h}^{-1} \times 65 \text{ kg}} = h$$

(Note that the units on the top and bottom cancel each other out i.e. kcal on bottom cancels out kcal on the top, and kg$^{-1}$ and kg cancel each other out on the bottom).

$$h = \frac{300}{195} = \text{approximately 1.5 hours.}$$

### 8.1.4 The EM spectrum: wavelength, frequency and speed

In Topic 5 Section 3.3.3 you will learn about an equation that describes the relationship between the wavelength and frequency of light (and other forms of electromagnetic radiation):

speed = wavelength × frequency

The speed of all light in a vacuum or in air can be regarded as a constant value: $3 \times 10^8$ m s$^{-1}$ so the characteristics of light from different parts of the electromagnetic (EM) spectrum are determined by the differing wavelength and frequency.

In Topic 8 you will learn how to calculate the wavelength of visible light of a given frequency. To work out the wavelength of blue-green light, which is roughly in the middle of the visible part of the spectrum, you need to know that its frequency is approximately $6 \times 10^{14}$ Hz. First we need to start by rearranging the equation to make wavelength the subject. Starting with:

speed = wavelength × frequency

Divide both sides of the equation by frequency to leave wavelength on its own:

$$\frac{\text{speed}}{\text{frequency}} = \text{wavelength}$$

Then, for convenience, swap the equation round to place wavelength on the left, making it the subject of the equation:

$$\text{wavelength} = \frac{\text{speed}}{\text{frequency}}$$

Inserting the values for the speed of light and the frequency of blue-green light into this equation gives:

$$\text{wavelength} = \frac{3 \times 10^8 \text{ m s}^{-1}}{6 \times 10^{14} \text{ Hz}}$$

This equation can be simplified by dividing separately the number and the 'power of ten' components of each term (Section 6.3.1). Note that you can also simplify the units if you remember that Hz = s$^{-1}$, so you can cancel the s$^{-1}$ on the top and bottom of the equation to give:

$$\text{wavelength} = \frac{3 \times 10^8}{6 \times 10^{14}} \text{ m}$$

Now you can separately divide the numbers and the powers of ten (Section 6.3.1):

$$\text{wavelength} = \frac{3}{6} \times \frac{10^8}{10^{14}} \text{ m}$$

3/6 is the same as ½, or 0.5 in decimal notation, so we can write the equation as:

$$\text{wavelength} = 0.5 \times \frac{10^8}{10^{14}} \text{ m}$$

Remember that to divide powers of ten, you can *subtract* the power of ten on the bottom of the fraction from the power of ten on the top, so

$$0.5 \times \frac{10^8}{10^{14}} \text{ m} = 0.5 \times 10^{(8-14)} \text{ m} = 0.5 \times 10^{-6} \text{ m}$$

but following the rules for scientific notation (the multiplier in a power of ten has to be greater than or equal to 1 and less than 10, Section 6.1) this should be written as $5 \times 10^{-7}$ m.

Therefore the wavelength of blue-green light is $5 \times 10^{-7}$ m

A scientific calculator can be used in scientific notation mode to do this part of the calculation in one step. For example, if your calculator uses an EXP key for powers of ten (see section 6.3.3) then you would need to key in 3 EXP 8 ÷ 6 EXP 14 = to get the answer as $5 \times 10^{-7}$.

Because the wavelength of visible light is so small, it is conventionally given in nanometres (nm), not metres. One nanometre is equal to $10^{-9}$ metre (Section 3.1), so to convert from metres (m) into nanometres ($10^{-9}$ m), you need to multiply by $10^9$.

Remember, to multiply one power of ten by another, you just add together the individual powers, for example, $10^3 \times 10^2 = 10^{(3+2)} = 10^5 = 100\,000$

Therefore for blue-green light:

wavelength = $(5 \times 10^{-7}) \times 10^9$ nm $= 5 \times 10^{(-7+9)}$ nm

wavelength = $5 \times 10^2$ nm $= 5 \times 100$ nm

wavelength = 500 nm

The wavelength of blue-green light is 500 nm.

### 8.1.5 The focusing ability of a lens: optical power

In Topic 8 you will learn that a lens is able to correct vision by refracting light rays so that they converge on the retina of the eye. The extent to which a lens can refract light is called the optical power, and is measured in units called dioptres [dye-op-ters] (D).

Changing the curvature of a convex lens (by changing the thickness in the centre) changes the focal length. A thin lens has a long focal length, and forms a sharp image a long distance away from the lens. A thick lens has a short focal length and forms a sharp image nearer to the lens. This thick lens is referred to commonly as a *powerful* lens, or a lens with a high optical power; the thin lens is a *weak* lens, with low optical power.

The optical power (in dioptres, D) of a lens is calculated using the equation:

$$\text{optical power (D)} = \frac{1}{\text{focal length (m)}}$$

Note that in this equation, focal length is in metres.

- Use Equation 5.1 above to calculate the optical power of a convex lens with a 10 cm (centimetre) focal length.

  - 10 cm is 0.10 m so the optical power in dioptres (D) is 1 divided by 0.10 = 10 D.

To determine the focal length of a lens if you know its optical power the equation can be rearranged by multiplying both sides by focal length and dividing both sides by optical power:

$$\text{optical power (D)} = \frac{1}{\text{focal length (m)}}$$

can therefore be rearranged to:

$$\text{focal length (m)} = \frac{1}{\text{optical power (D)}}$$

- Use this equation to calculate the focal length in centimetres of a convex lens with an optical power of 7 D.

  - Focal length in metres is 1 divided by 7 D = 0.1428 m = 14 cm to two significant figures.

**Practice questions 15**

(a) The equation for the volume of a box is $v = l \times h \times w$. If a box has a volume of 450 cm³, a length of 15 cm and width of 10cm, what is its height?

(b) Calculate the BMI of a person with body mass 72 kg and height 175 cm (giving your answer to an appropriate number of significant figures) using the equation:

$$\text{BMI} = \frac{\text{mass (kg)}}{(\text{height m})^2}$$

(c) If jogging has a value of 7 METs, i.e. 7 kilocalorie per kilogram of body mass per hour of exercise (7 kcal kg⁻¹ h⁻¹), how long would it take a person weighing 68 kg to expend 500 kcals by jogging?

(d) If yellow light has a wavelength of 570 nm and the speed of light in air is $3 \times 10^8$ m s⁻¹, calculate the frequency of the yellow light in s⁻¹ (Hz) using the following equation:

speed = wavelength × frequency

(e) Using the following equation, calculate the focal length in centimetres of a convex lens with an optical power of 5D.

$$\text{optical power (D)} = \frac{1}{\text{focal length (m)}}$$

# 9. Tables, graphs and charts

Tables, graphs and charts are a way of summarising and presenting a large amount of data in a form that allows the reader to absorb the information and draw conclusions quickly. This section briefly describes the characteristics of the types of tables, charts and graphs you will meet in the module, and discusses where it is appropriate to use them.

# 9.1 Tables

Tables are an excellent way of making all sorts of information and data much easier to understand and use. Tables consist of horizontal rows and vertical columns – often the first row and first column contain headings describing the type of information. For example Table 7 displays definitions and examples of the different levels of organisation in the human body (from Topic 2 Section 2.4).

**Table 7** Organisation in the human body

| Level of organisation | Definition | Examples in the human body |
|---|---|---|
| **organism** | a form of life | the entire human body |
| **body system** | a group of organs that function together as a unit | the digestive system; the cardiovascular system; the nervous system |
| **organ** | a distinct structure composed of different tissues which together perform a specialised task | the stomach, the liver, the heart, the brain |
| **tissue** | a group of cells with similar structure and function | epithelial tissue, muscle tissue, nerve tissue, connective tissue |
| **cell** | the smallest unit of an organism | muscle cells, erythrocytes, adipocytes |
| **organelle** | a structure within a cell that is bounded by a membrane | nucleus, mitochondrion, endoplasmic reticulum |
| **molecule** | two or more atoms held together by covalent chemical bonds | proteins, fats, carbohydrates, water |

Tables are also a versatile way of displaying a large amount of numerical data. Table 8 (from Topic 2 Section 1.4) shows the top ten leading causes of death in 2012. Note that the headings above columns three to five state the unit of measurement for the numbers in that column. Putting the units in the column headings means that the unit does not have to be repeated after each number in the column. The column heading for the total number of deaths in column three ends in thousands ('000s') to indicate that you must multiply the numbers in that column by 1000 (or simply add 000 to the end of each number); abbreviating big numbers in this way often occurs in numerical tables to avoid having to repeat a lot of zeros at the end of every number. In column four the numbers represent the percentage of total deaths attributable to each cause and column five gives the mortality rate – the number of deaths that occurred in every 100 000 people in the global population for each of the causes.

**Table 8** Top ten leading causes of death worldwide in 2012.

| Rank | Cause | Total deaths (000s) | % of total deaths | Mortality rate per 100 000 population |
|---|---|---|---|---|
| 0 | All causes | 55 859 | 100.0 | 789.5 |

| | | | | |
|---|---|---|---|---|
| 1 | Coronary heart disease (caused by blockage of blood vessels supplying the heart) | 7356 | 13.2 | 104.0 |
| 2 | Stroke | 6671 | 11.9 | 94.3 |
| 3 | Chronic obstructive pulmonary disease | 3104 | 5.6 | 43.9 |
| 4 | Lower respiratory infections | 3052 | 5.5 | 43.1 |
| 5 | Trachea, bronchus and lung cancers | 1600 | 2.9 | 22.6 |
| 6 | HIV/AIDS | 1534 | 2.8 | 21.7 |
| 7 | Diarrhoeal diseases | 1498 | 2.7 | 21.2 |
| 8 | Diabetes | 1497 | 2.7 | 21.2 |
| 9 | Road injury | 1255 | 2.3 | 17.7 |
| 10 | Hypertensive heart disease (caused by high blood pressure) | 1141 | 2.0 | 16.1 |

- From Table 8, write out in words the total number of deaths from lower respiratory infections.
- The total number of deaths from respiratory infections is 3 052 000; three million, fifty two thousand.
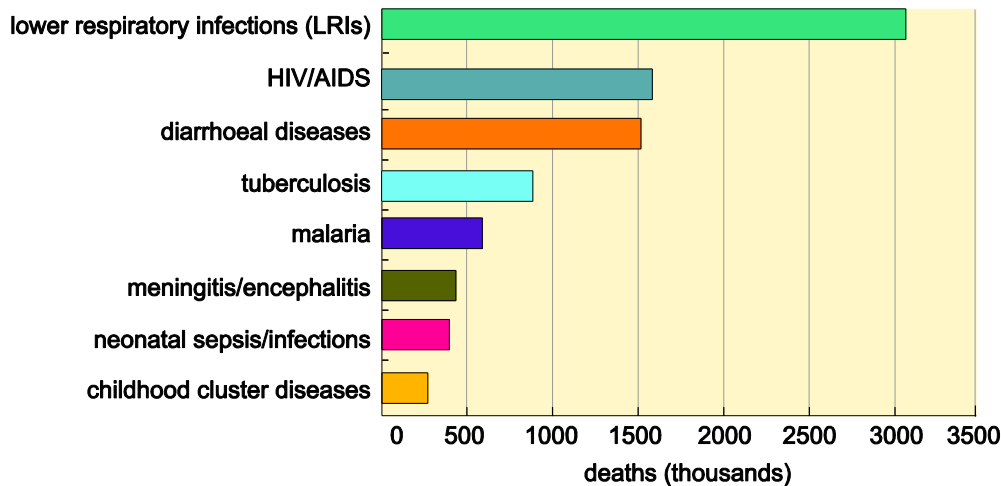
## 9.2 Bar charts

Bar charts are used to display data that falls into distinct groups or categories, for example countries, blood groups or types of disease. The measurement related to each category is represented by a bar. The height or length of the bar is proportional to the value of the measurement and can be read from a scale on the adjacent axis. Figure 2 shows a bar chart (from Topic 2, Section 1.3.8) displaying the number of obese people in ten countries.
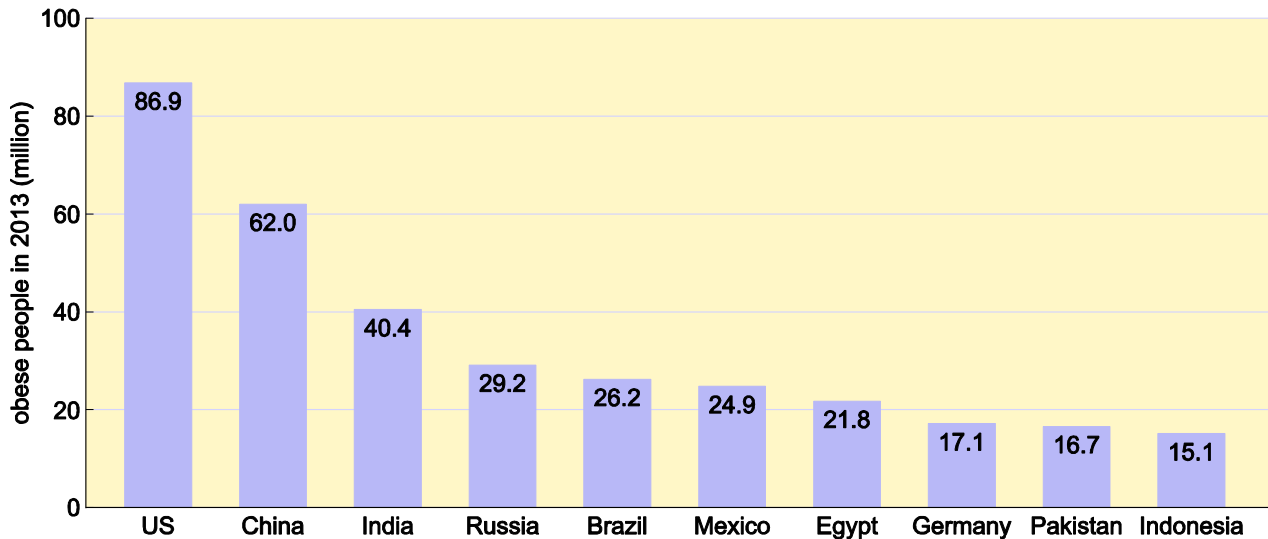
**Figure 2** Vertical bar chart showing the ten countries with the largest numbers of obese people.

Bar charts may have the bars stacked either vertically, like Figure 2, or horizontally like Figure 3 (from Topic 1, Section 4.1.3). The choice of orientation depends on the most convenient layout – horizontal bar charts often make it easier to read the labels for the categories represented by each bar.



**Figure 3** Horizontal bar chart showing the number of worldwide deaths (in thousands) in 2012 due to eight major categories of infectious diseases.

## 9.3 Histograms

Histograms are very similar to bar charts, but the important difference is that instead of distinct categories, they display a continuous series of data, for example, a range of heights or ages. Their purpose is to show the distribution of the data across the range of values. Figure 4 shows a population pyramid for the UK in 2010 (from Topic 1, Section 5.5). It is composed of two horizontal histograms, side by side, showing how many people there are in each age range represented along the vertical axis – with males shown in the histogram on the left and females shown in the histogram on the right. Notice that the vertical axis represents a continuous increase in age, divided into age ranges of 5 years for each bar. This population pyramid makes it easy to see at a glance the age structure of the UK in that year.
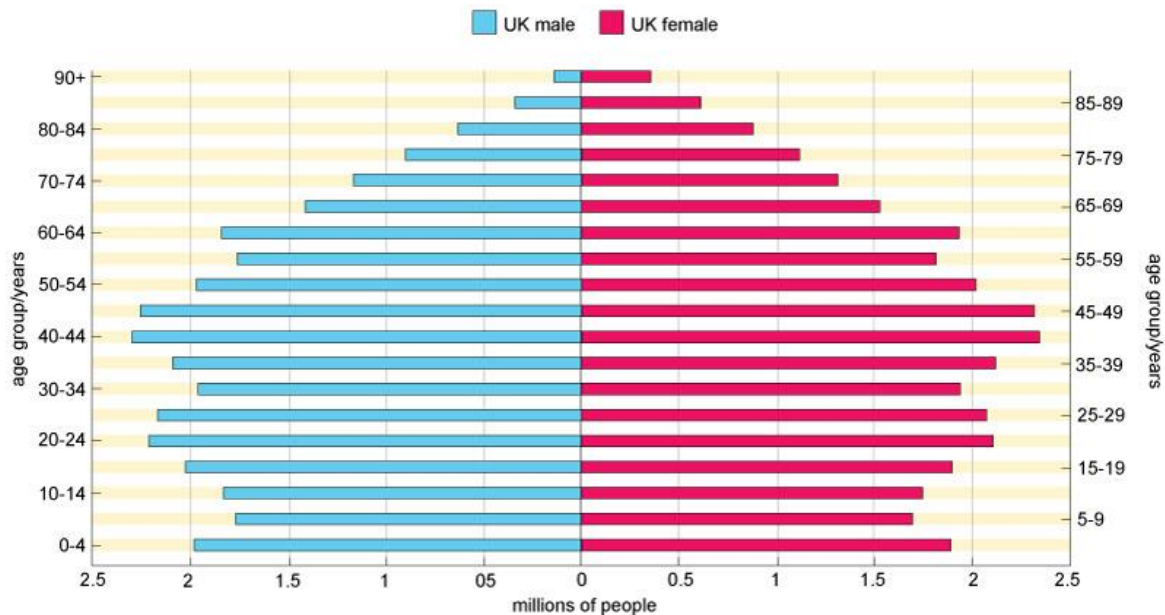
**Figure 4** Population pyramid for the UK in 2010.

- From the shape of the population pyramid in Figure 4, how would you describe the age structure of the UK?
- The pyramid is narrow at the bottom showing that there are relatively few younger people and wider in the middle because there are larger numbers of people in the middle age ranges. It then gradually narrows near the top because relatively few people survive into very old age.

### 9.3.1 Distributions and averages

Numerical data for a group or population can show various different sorts of distributions. The type of distribution shown in Figure 5a in which the individual values are evenly spread either side of a central value or average is called a 'normal distribution' (sometimes referred to as a 'bell curve' because it is shaped like a bell). In contrast, the data in Figure 5b is skewed towards higher values, while that in Figure 5c is randomly distributed.



(a) (b) (c)
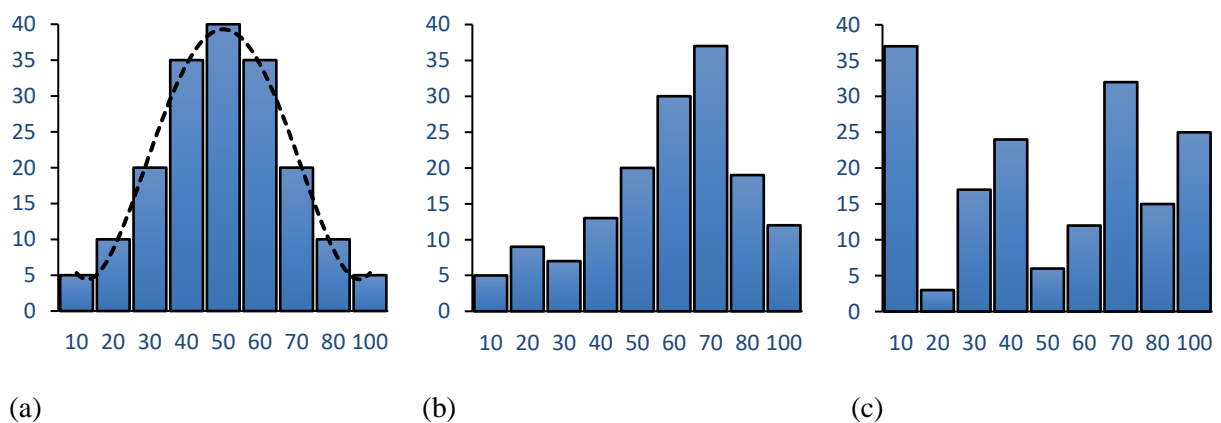
**Figure 5** The distribution of different types of data (a) normal distribution, (b) skewed distribution and (c) random distribution.

The average value for a set of data can be measured in different ways.

- The *mean* value is the type of average you will use most often. The mean is the average of all the values in a set of data. To find a mean you simply add all the values together then divide by the total number of values.

- The *median* value is the middle number in a range of values – in other words if all the values are ranked in order of size, the median would be the value that is equidistant from each end of the list.

- The *mode* is simply the value that occurs most often. If no value is repeated, then there is no mode for the list.

For the following range of values:

<div align="center">13, 17, 13, 16, 12, 13, 16, 20, 18, 13, 14</div>

- The mean is the average of all the values, so:
  (13 + 17 + 13 + 16 + 12 + 13 + 16 + 20 + 18 + 13 + 14) ÷ 11 values = 15

- The median is the middle value, so if you write the list in order:

  12, 13, 13, 13, 13, <u>14</u>, 16, 16, 17, 18, 20

  the median is the middle value in the range, which is 14

- The mode is the value that appears most often, which in this case is 13.

## 9.4 Pie charts

A pie chart is a visually appealing way to display categorical data in which each category is a 'slice' of the pie representing a proportion, fraction or percentage of a whole. Using a pie chart for data that doesn't add up to a whole (i.e. 100%) can be misleading. Figure 6 shows the proportional mortality from different types of infectious disease (from Topic 1, Section 4.2.1). Note that the numbers add up to 100% because an extra category, 'other infections' has been included. Pie charts usually work best where there is a relatively small number of categories. The sections of the pie are often represented in different colours.
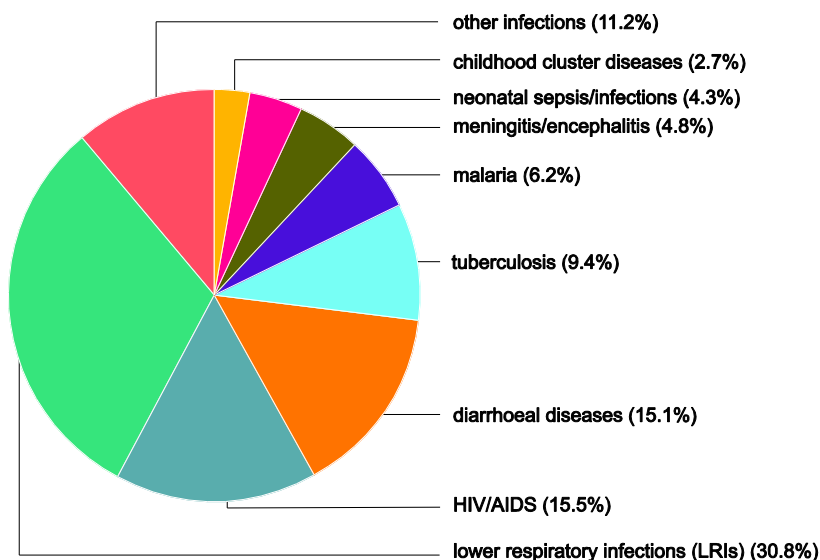


**Figure 6** Pie chart showing the proportional mortality from the eight categories of infectious diseases causing the most deaths worldwide in 2012, plus all 'other infections'.

## 9.5 Line graphs

A line graph is used to show data that is connected in a continuous way. Line graphs are particularly suited to showing time trends i.e. changes in a particular variable over a period of time. Figure 7 (from Topic 2, Section 4.5.1) shows the incidence of leprosy in three countries and worldwide from 2005 to 2012. Each data point represents the value of the variable (in this case the incidence of leprosy) in the year the measurement was taken. Line graphs are named after the lines that join the data points on the graph, making it easier to see how the two variables represented in the graph relate to each other.

There is a convention in graph plotting that if one of the variables is 'time', the time scale is plotted on the horizontal (x) axis, and the scale for the other variable is plotted on the vertical (y) axis. Another

convention is that usually the values on the horizontal x axis increase from left to right, while the values on the vertical y axis increase from bottom to top. The vertical and the horizontal axes are marked with 'ticks' that show the location of each numbered increment along the scales.
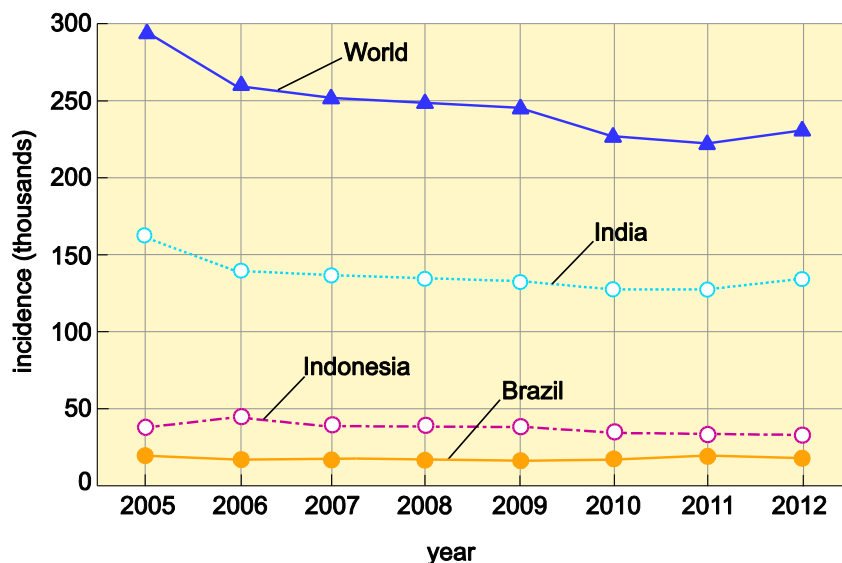


**Figure 7** Line graph showing the trend in leprosy incidence (thousands of new cases per year) from 2005–2012 in India, Indonesia and Brazil and the global total.

- From Figure 7, what was the incidence of leprosy in India in 2011?
- To read this value, locate 2011 on the horizontal x axis then follow the vertical line upwards to locate the data point for India, then follow an imaginary line across to the vertical y axis. You can use the scale to read off the incidence of leprosy in India in 2011 as about 130 000.

Figure 8 shows another line graph displaying infant mortality rates in England and Wales and in Stoke-on-Trent between 1855 and 1880 (from Topic 1, Section 5.4.5). Notice that the vertical y axis on this graph is interrupted – it starts at 140 infant deaths per 1000 live births. (The break near the bottom of the scale signifies where it has been interrupted and nothing below that point is shown).
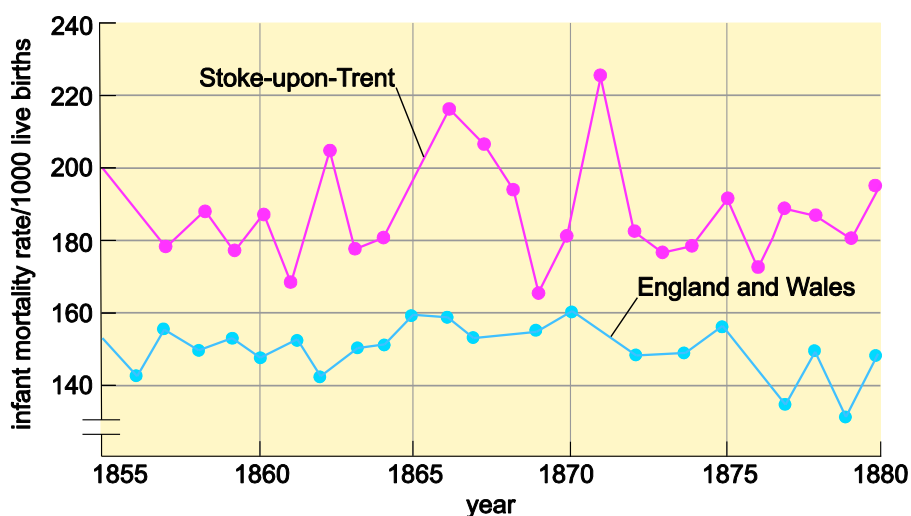


**Figure 8** Infant mortality rates (deaths under 1 year of age) per 1000 live births in England and Wales and Stoke-upon-Trent between 1855 and 1880.

The infant mortality rate was always much higher than zero throughout the period, so if the vertical scale had begun at zero there would be a lot of unused space across the bottom of the graph and the data would have been squashed up at the top. Starting higher up the scale spreads the data out making it easier to see the relatively small changes in infant mortality from year to year. Be careful when interpreting graphs that don't start at zero, as in some cases it may inadvertently exaggerate the apparent changes in a variable over time. Note that the scale on the horizontal axis does not start at zero either, because the data don't go back to the 'year zero'.

# 9.6 XY graphs (scatter plots)

XY graphs, commonly called scatter plots, are similar to line graphs in that the data points display the relationship between two variables. The difference between line graphs and scatter plots is that in a scatter plot the data points are not connected to each other by straight lines. Instead the graph expresses a trend - a general direction in which something is developing or changing.

Figure 9 shows a scatter plot (from Topic 1, Section 5.4.5) displaying the relationship between mortality rates for children and access to improved sanitation in a number of countries. Conventionally the 'independent variable' (the variable determined in advance, in this case the level of access to sanitation in each country) is plotted on the horizontal x axis, while the 'dependent variable' (the variable being measured, in this case the child mortality rate) is plotted on the vertical y axis.
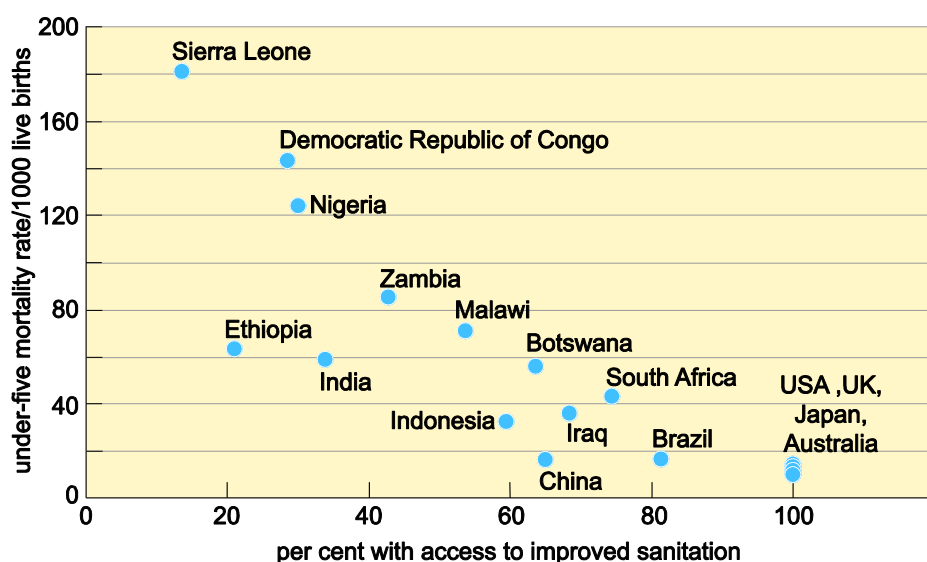


**Figure 9** Scatter diagram showing the association between the age-specific mortality rates for children aged under five years (per 1000 live births) in selected countries in 2011–12 and percentage of population with access to improved sanitation.

Trying to connect together all the data points in Figure 8 with straight lines wouldn't be very helpful. In fact it would quickly obscure the fairly obvious relationship between these two variables. To make this relationship even clearer, a line of best fit through the data points - known as a trend line or a regression line, could be added to the scatter plot.

- If you were to add, just by eye, a trend line to Figure 9, based on the distribution of the data points, which way would the line slope, and how would you describe the trend it shows in the association between child mortality and access to improved sanitation?
- The trend line would slope downwards from left to right, indicating that child mortality *decreases* as the percentage of the population with access to improved sanitation *increases*.

The scatter plot in Figure 9 includes a lot of data points, but makes it immediately clear that there is a negative association between child mortality and access to improved sanitation, because as one of these variables increases, the other decreases.

## 9.7 The types of association between two variables

Three patterns are commonly observed in graphs showing the relationship between two variables:

- there is a positive association between the two variables if the value of one variable increases as the value of the other variable increases (Figure 10a).

- there is a negative (or inverse) association between two variables if the value of one variable decreases as the value of the other variable increases (Figure 10b)

- there is no association between two variables if there is no consistent pattern of change in one variable as the other variable changes (Figure 10c).
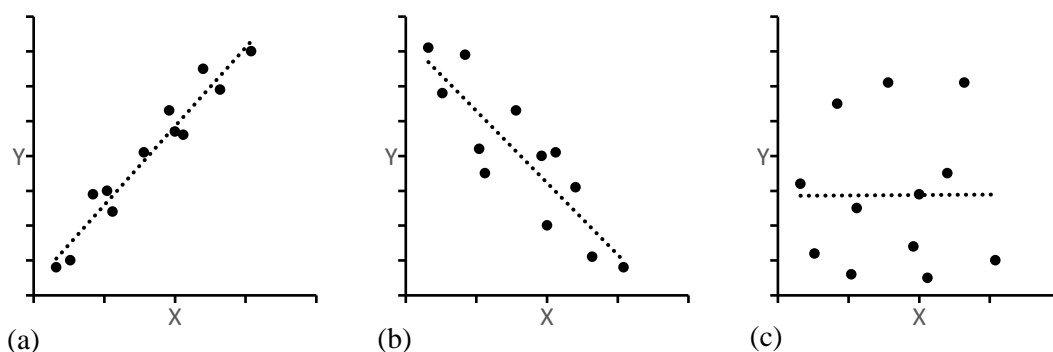


**Figure 10** Three graphs showing (a) a positive association, (b) a negative association and (c) no association between two variables.

As well as these linear associations, there are other common patterns of association represented by curved or s-shaped (sigmoidal) trend lines, but these are beyond the scope of this workbook.

An important point to bear in mind is that an association (or correlation) seen in graphed data simply shows that the variables change in relation to each other in a particular way. Such an association does not necessarily prove that the changes in one variable directly *cause* the change in the other variable – there may be other factors that influence the change in both variables.

**Practice questions 16**

Decide whether the following types of data would be displayed most appropriately in a pie chart, a bar chart, a line graph or a scatter plot.

(a) The incidence of tuberculosis in the population of ten different cities.

(b) The increase in diabetes incidence in the UK over the last thirty years.

(c) The proportions of a population who are sedentary, moderately physically active and very physically active.

(d) Gross domestic product (GDP) and the average age of death in fifty countries.

# 10. And finally… rate your own maths skills

Table 9 lists the maths skills developed in this module so that you can see how much you have learned (these are just the maths skills – you have learned much else too). You may find it helpful at the beginning and the end of the module to rate your own ability in each of these skills by ticking the appropriate box in each row of the table. In judging your ability, you could think about whether you were able to do the relevant questions in this workbook. It doesn't matter if you couldn't do them, or if you found them hard – did you manage to do them in the end? Also, how well did you get on with maths questions in the module? What comments did you get from your tutor about maths in your assignments? If you feel you are lacking confidence in any of the mathematical skills taught in this course there are some more resources available on StudentHome at http://www2.open.ac.uk/students/help/if-you-get-stuck-with-maths.

**Table 9** Your maths skills

| Maths skill | Confident | Fairly confident | Still struggling | Major problem |
|---|---|---|---|---|
| Using correct order in calculations (BEDMAS) | | | | |
| Using negative numbers | | | | |
| Using powers | | | | |
| Using fractions and percentages | | | | |
| Using ratios | | | | |
| Using SI units of measurement | | | | |
| Using prefixes with SI units | | | | |
| Using decimal numbers | | | | |
| Using decimal places and rounding | | | | |
| Using powers of ten and scientific notation | | | | |
| Using and rearranging equations | | | | |
| Using significant figures | | | | |
| Reading information from a table | | | | |
| Interpreting graphs and charts | | | | |
| Constructing a table of data | | | | |
| Plotting graphs and charts | | | | |

# 11. Answers to practice question

**Practice questions 1**

Do the following calculations without using your calculator. Then use your calculator to check the answers.

(a) $3 \times 4 + 2$          Answer: 14

(b) $2 + 4 \times 3$          Answer: 14 (remember to do multiplication and division first)

(c) $4 \times 2 - 21 \div 7$          Answer: 5

(d) $7 + 9 \div 2$          Answer: 11.5

**Practice questions 2**

Calculate the following, giving the answer as a decimal number.

(a) $3^3$          Answer: $3 \times 3 \times 3 = 27$

(b) $3 \times 5^2$          Answer: $3 \times 5 \times 5 = 75$

(c) $3^2 + 4^2$          Answer: $(3 \times 3) + (4 \times 4) = 9 + 16 = 25$

(d) $12^2 \div 3^2$          Answer: $(12 \times 12) \div (3 \times 3) = 144 \div 9 = 16$

(e) $2^{-2}$          Answer: $\frac{1}{2 \times 2} = \frac{1}{4} = 0.25$

(f) $2 \times 4^{-2}$          Answer: $2 \times \frac{1}{4 \times 4} = \frac{2}{16} = 0.125$

**Practice questions 3**

To practise the BEDMAS rules, try the following calculations.

(a) $150 \div 10 \times 3$            Answer: 45

(b) $150 \div (10 \times 3)$         Answer: 5

(c) $(2 \times 3)^2$               Answer: 36

(d) $2 \times 3^2$                Answer: 18

(e) $(10 - 5) \times (3 + 1) + 4$     Answer: 24

(f) $(3 + 2)^2 - (7 - 4)^2$      Answer: 16


**Practice questions 4**

Try the following calculations without using your calculator:

(a) $3 - 6$                 Answer: –3

(b) $-4 - (-2)$           Answer: –2

(c) $5 + (-1)$             Answer: 4

(d) $-7 + 4$              Answer: –3

(e) $-7 \times 5$             Answer: –35

(f) $4 \times (-3)$           Answer: –12

(g) $8 \div (-2)$           Answer: –4

(h) $-12 \div (-3)$        Answer: 4


**Practice questions 5**

Carry out the following conversions, including the correct unit symbol in your answer.

(a) The number of joules in 5 kilojoules         Answer: 5000 J

(b) The number of milliseconds in 2 seconds     Answer: 2000 ms

(c) The number of centimetres in 6.6 metres     Answer: 660 cm

(d) The number of micrograms in 2.2 grams     Answer: 2 200 000 µg

(Note: µ is the greek letter 'mu' which you will find in the symbol font in your word processor programme)

(e) The number of nanometres in 3 millimetres     Answer: 3 000 000 nm

(f) What is 250 grams in kilograms?           Answer: 0.25 kg

(g) What is 600 millimetres in metres?         Answer: 0.6 m

(h) What is 3500 micrograms in grams?        Answer: 0.0035 g

## Practice questions 6

Try the following calculations:

(a) If a nerve signal travels 0.1 metre in 1 millisecond, what is its speed in $m\,s^{-1}$?

Answer: $0.1\ m \div 1\ ms = 0.1\ m \div 0.001\ s = 100\ m\,s^{-1}$

(b) The labelling of a standard cola drink indicates that it contains 10.6 g of sugar per 100ml, while a lemonade drink contains 22.5g of sugar per 250 ml. What is the concentration of sugar in $g\,l^{-1}$ in each type of drink?

Answer: The cola drink contains $106\ g\,l^{-1}$ and the lemonade drink contains $90\ g\,l^{-1}$

## Practice questions 7

Calculate the following:

(a) $\frac{2}{5}$ of 20                    Answer: 8

(b) $\frac{7}{8}$ of 24                    Answer: 21

(c) 15% of 60                    Answer: 9

(d) 60% of 5500                    Answer: 3300

(e) 3 as a percentage of 600                    Answer 0.5%

(f) 12 166 as a percentage of 55 300                    Answer: 22%

(g) A glass of beer contains a volume of 500 millilitres (ml). If the beer is 5% pure alcohol by volume, how many millilitres of pure alcohol are there in the glass?

Answer: 25 ml

## Practice questions 8

In a group of Open University students, 16 were men and 8 were women.

(a) What was the ratio of women to men in the group?

Answer: 8:16 which can be simplified to 1:2

(b) What fraction of the total group were women?

Answer: $\frac{8}{24}$ which can be simplified to $\frac{1}{3}$

## Practice questions 9

(a) Write a place-value table for the decimal number 56.739

Answer:

| Tens | Units | Tenths | Hundredths | Thousandths |
|------|-------|--------|------------|-------------|
| 5    | 6     | 7      | 3          | 9           |

(b) Convert the following fractions to decimal numbers: $\frac{1}{5}$; $\frac{3}{16}$ and $\frac{24}{25}$

Answer: 0.2; 0.1875 and 0.96.

## Practice questions 10

Round each of the following numbers to one decimal place, to two decimal places, and to three decimal places.

(a) 1.2648          Answers: 1; 1.3; 1.26; 1.265

(b) 0.8255          Answers: 1; 0.8; 0.83; 0.826

(c) 21.9184         Answers: 22; 21.9; 21.92; 21.918

## Practice questions 11

Express the following numbers in scientific notation.

(a) 100 000 000          Answer: $1 \times 10^8$

(b) 35 000          Answer: $3.5 \times 10^4$

(c) $95 \times 10^5$          Answer: $9.5 \times 10^6$

(d) $0.51 \times 10^3$          Answer: $5.1 \times 10^2$

Convert the following scientific notation numbers into decimal form:

(e) $7.3 \times 10^4$          Answer: 73 000

(f) $4.44 \times 10^5$          Answer: 444 000

(g) $6.05 \times 10^3$          Answer: 6050

## Practice questions 12

Write out the following numbers using scientific notation

(a)  0.00354          Answer: $3.54 \times 10^{-3}$

(b)  0.08274          Answer: $8.274 \times 10^{-2}$

(c)  0.00000224          Answer: $2.24 \times 10^{-6}$

Convert the following scientific notation numbers into decimal form.

(d)  $5.64 \times 10^{-3}$          Answer: 0.00564

(e)  $1.673 \times 10^{-5}$          Answer: 0.00001673

(f)  $2.3387 \times 10^{-2}$          Answer: 0.023387

## Practice questions 13

Calculate the following giving your answer in scientific notation:

(a)  $(1 \times 10^4) \times (1 \times 10^3)$          Answer: $1 \times 10^7$

(b)  $10^2 \times 10^5$          Answer: $1 \times 10^7$

(c)  $(2.5 \times 10^3) \times (4 \times 10^2)$          Answer: $1 \times 10^6$

(d)  $(1 \times 10^6) \div (1 \times 10^3)$          Answer: $1 \times 10^3$

(e)  $10^8 \div 10^6$          Answer: $1 \times 10^2$

(f)   $(3.5 \times 10^8) \div (5 \times 10^3)$          Answer: $7 \times 10^4$

## Practice questions 14

How many significant figures are there in the following numbers?

(a) 5644                                                    Answer: 4 significant figures

(b) 5600                                                    Answer: 2 significant figures

(c) 5.06                                                    Answer: 3 significant figures

Give the following numbers to the stated number of significant figures:

(d) 36373 to three significant figures.                    Answer: 36400

(e) 0.008248 to three significant figures                  Answer: 0.00825

Give the answer to the following calculation to two significant figures:

(f) 3.5 cm + 0.50 cm + 0.055 cm                            Answer: 4.1 cm

(because all the numbers used in the calculation have two significant figures)


## Practice questions 15

(a) The equation for the volume of a box is $v = l \times h \times w$. If a box has a volume of 450 cm³, a length of 15 cm and width of 10 cm, what is its height?

Answer: First rearrange the equation to $h = \dfrac{v}{w \times l}$ then $\dfrac{450 \text{ cm}^3}{15 \text{ cm} \times 10 \text{ cm}} = 3 \text{ cm}$

(b) Calculate the BMI of a person with body mass 72 kg and height 175 cm (giving your answer to an appropriate number of significant figures) using the equation:

$$BMI = \frac{mass\ (kg)}{height\ (m)^2}$$

Answer: 175 cm is 1.75 metres, so this person's BMI is:

$$BMI = \frac{72 \text{ kg}}{1.75 \text{ m} \times 1.75 \text{ m}} = \frac{72 \text{ kg}}{3.06 \text{ m}^2} = 23.5294 = 24 \text{ kg/m}^2 \text{ (to two significant figures)}.$$

(c) If jogging has a value of 7 METs, i.e. about 7 kilocalorie per kilogram of body mass per hour of exercise (7 kcal kg⁻¹ h⁻¹), how long would it take a person weighing 68 kg to expend 500 kcals by jogging?

Answer: 500 kcal = 7 kcal kg⁻¹ h⁻¹ × 68 kg × an unknown number of hours (h)

To rearrange the equation you can divide both sides by 7 kcal kg⁻¹ h⁻¹ and 68 kg:

$$h = \frac{500 \text{ kcal}}{7 \text{ kcal kg}^{-1} \text{ h}^{-1} \times 68 \text{ kg}} = \frac{500}{476} = 1.05, \text{ so just over an hour of jogging.}$$

(d) If yellow light has a wavelength of 570 nm and the speed of light is $3.0 \times 10^8$ m s⁻¹, calculate the frequency of the yellow light in s⁻¹ (Hz) using the following equation:

speed = wavelength × frequency

First, divide both sides of the equation by wavelength to leave frequency on its own:

$$frequency = \frac{speed}{wavelength}$$

Then you need to convert the wavelength to metres to use it in this equation. There are $10^9$ nanometres in a metre, so to convert 570 nm to metres you need to divide 570 by $10^9$:

$(5.7 \times 10^2) \div (1 \times 10^9) = 5.7 \times 10^{(2-9)} = 5.7 \times 10^{-7}$ m

Now insert the values you have been given into the equation:

$$\text{frequency} = \frac{3.0 \times 10^8 \text{ m s}^{-1}}{5.7 \times 10^{-7} \text{ m}}.0$$

or $\dfrac{3.0}{5.7} \times \dfrac{10^8}{10^{-7}}$ s$^{-1} = 0.53 \times 10^{8-(-7)}$ s$^{-1} = 0.53 \times 10^{15}$ s$^{-1} = 5.3 \times 10^{14}$ s$^{-1}$

So the frequency of yellow light is $5.3 \times 10^{14}$ s$^{-1}$ (or $5.3 \times 10^{14}$ Hz).

(e) Using the following equation, calculate the focal length in centimetres of a convex lens with an optical power of 5D.

$$\text{optical power (D)} = \frac{1}{\text{focal length (m)}}$$

Answer: First rearrange the equation to:

$$\text{focal length (m)} = \frac{1}{\text{optical power (D)}}$$

Focal length in metres is 1 divided by 5 D = 0.2 m. There are 100 cm in a metre, so the focal length is 20 cm.

## Practice questions 16

Decide whether the following types of data would be displayed most appropriately in a pie chart, a bar chart, a line graph or a scatter plot.

(a) The incidence of tuberculosis in the population of ten different cities.

Answer: This data is grouped by city, so the most appropriate way of displaying it would be a bar chart.

(b) The increase in diabetes incidence in the UK over the last thirty years.

Answer: This data represents continuous change over time, so a line graph may be most appropriate.

(c) The proportions of a population who are sedentary, moderately physically active and very physically active.

Answer: This data divides the whole population into three sectors, so a good way of showing it would be a pie chart.

(d) Gross domestic product (GDP) and the average age of death in fifty countries.

Answer: This data looks at the relationship between two variables. A scatter plot would be the best way of displaying a trend (i.e. whether there is a clear association between national income and lifespan).